Experimental Observation of Oceanic Forced Convection

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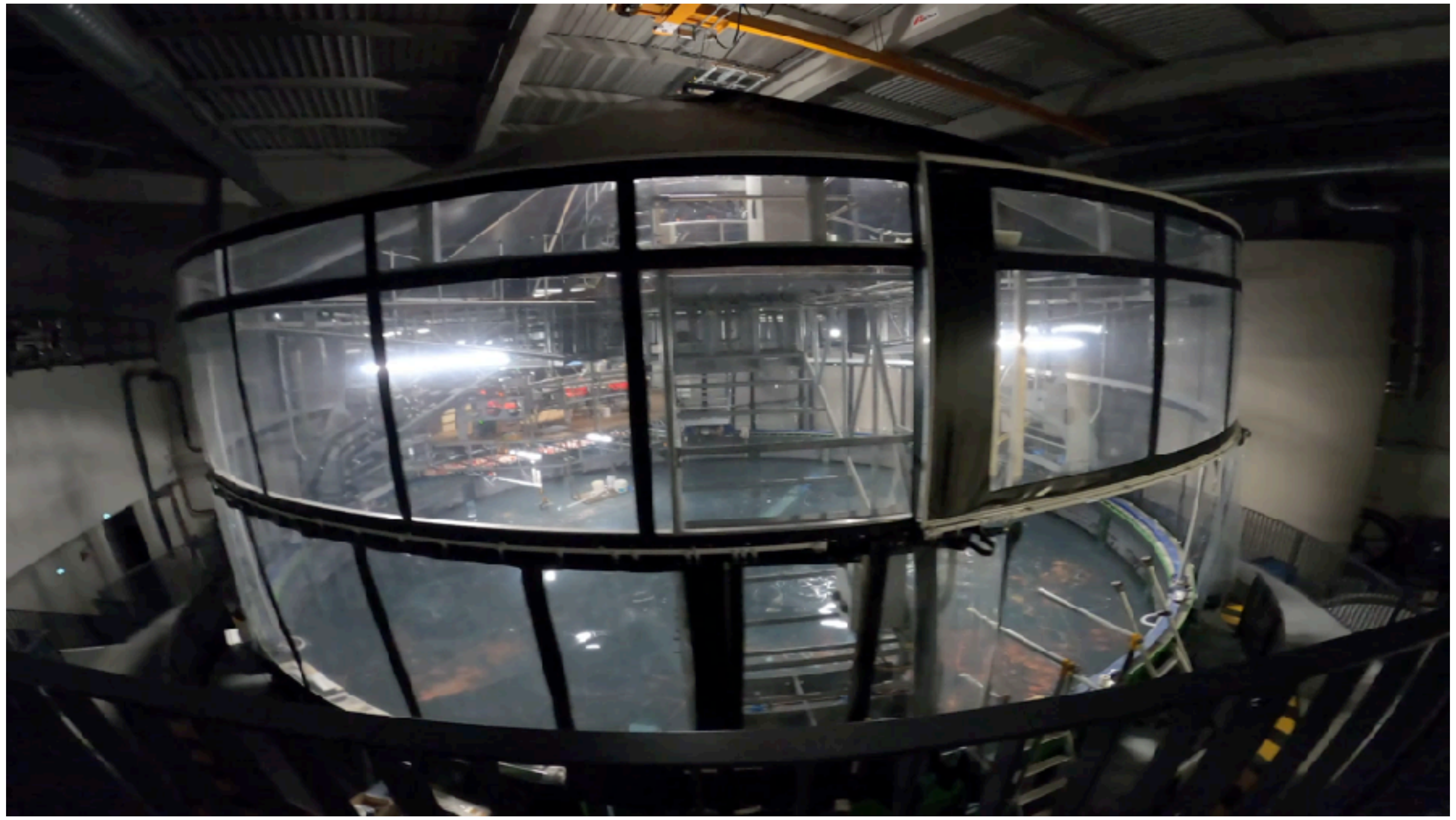
LABORATOIRE DES ECOULEMENTS GEOPHYSIQUES ET INDUSTRIELS

Institut des géosciences de l'environnement

Outline

- **Wind-driven surface dynamics**
  - With/without rotation
- **Experimental Mixed layer on Coriolis Plateform**
  - Apparatus
- **Vertical Structure of Entrainment**
  - Qualitative description
- **Experimental results**
  - Validation of the $k-\epsilon$ turbulent model closure



Forced Convection: Wind-driven circulation

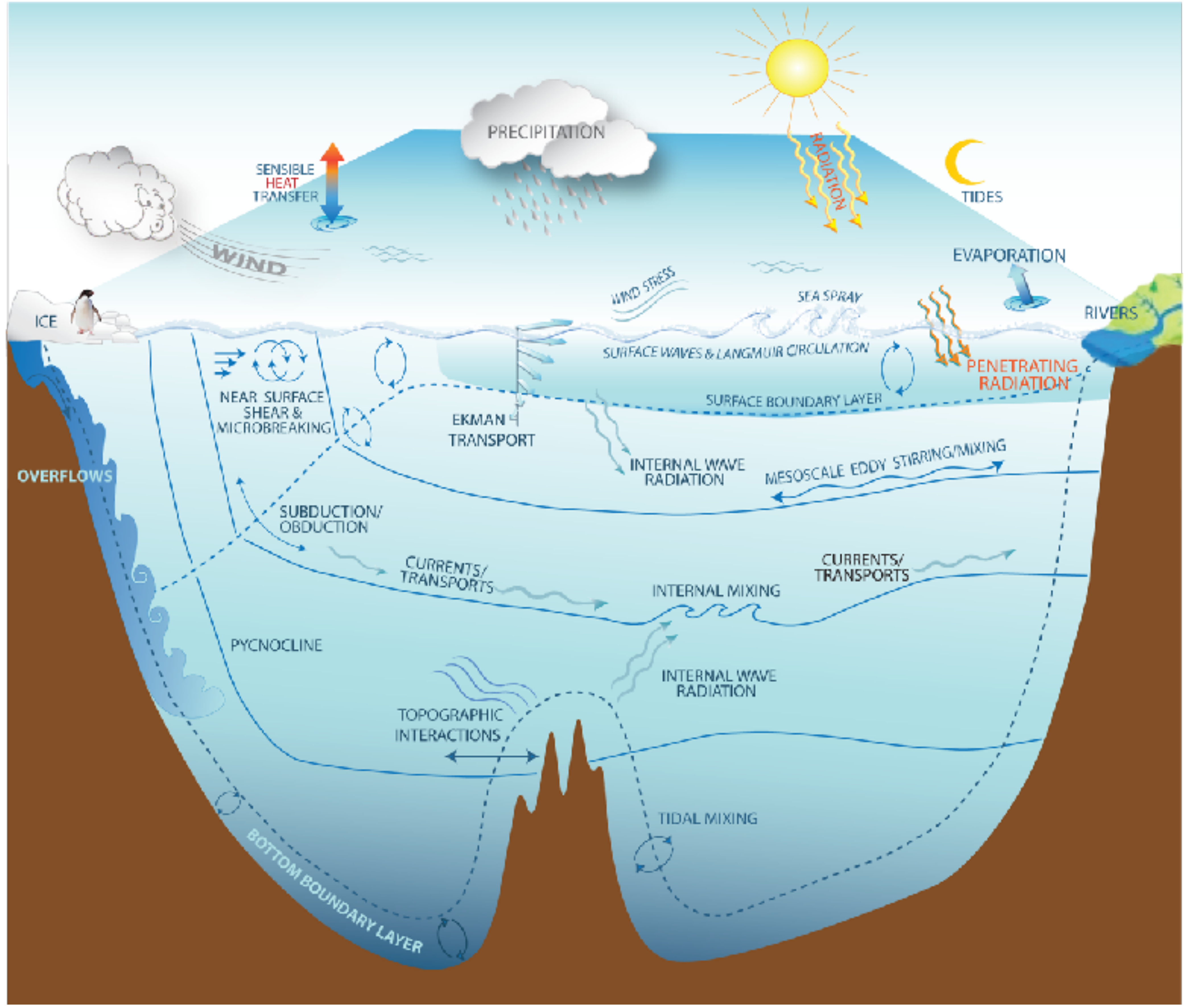


Figure from GFDL, NOOA

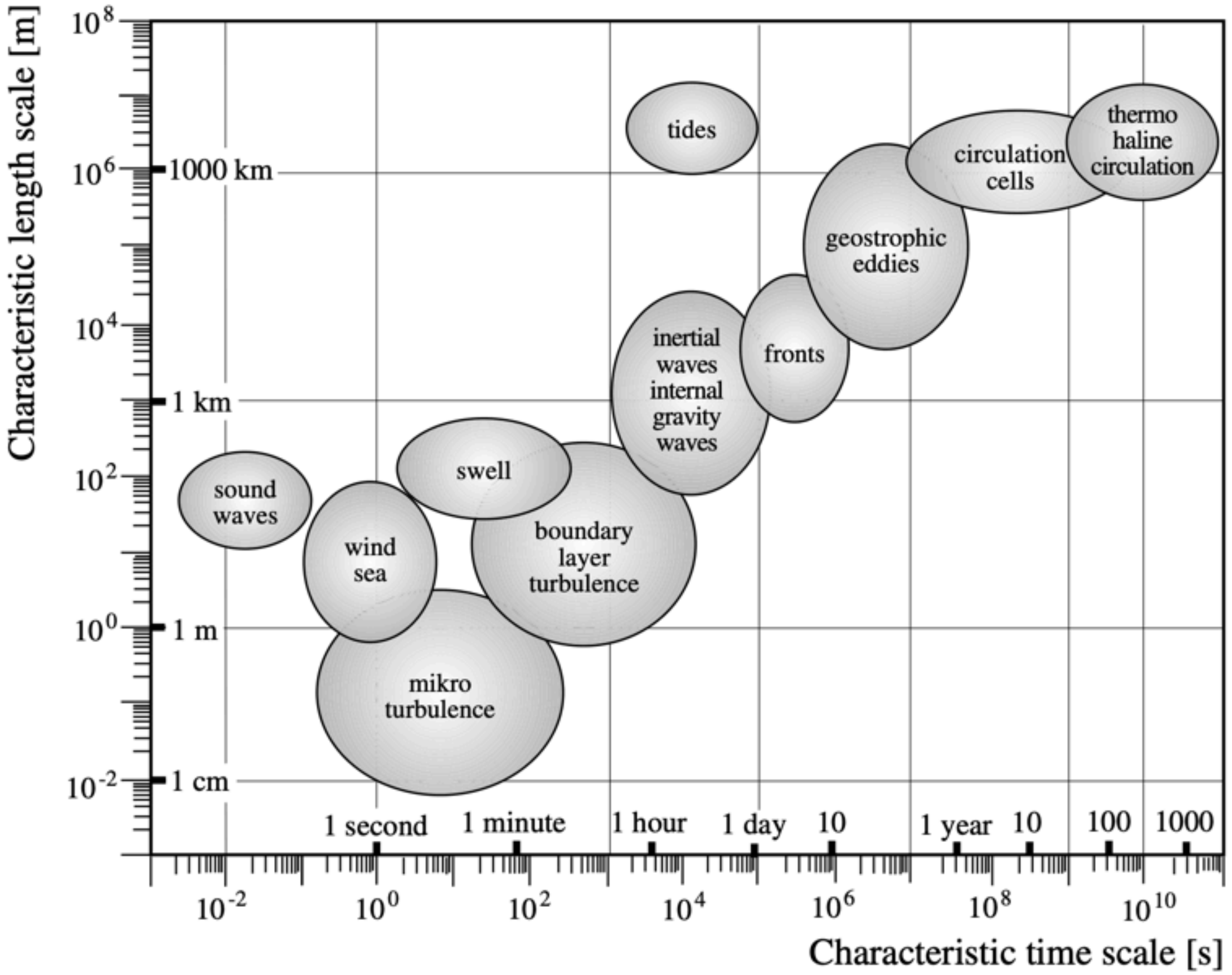
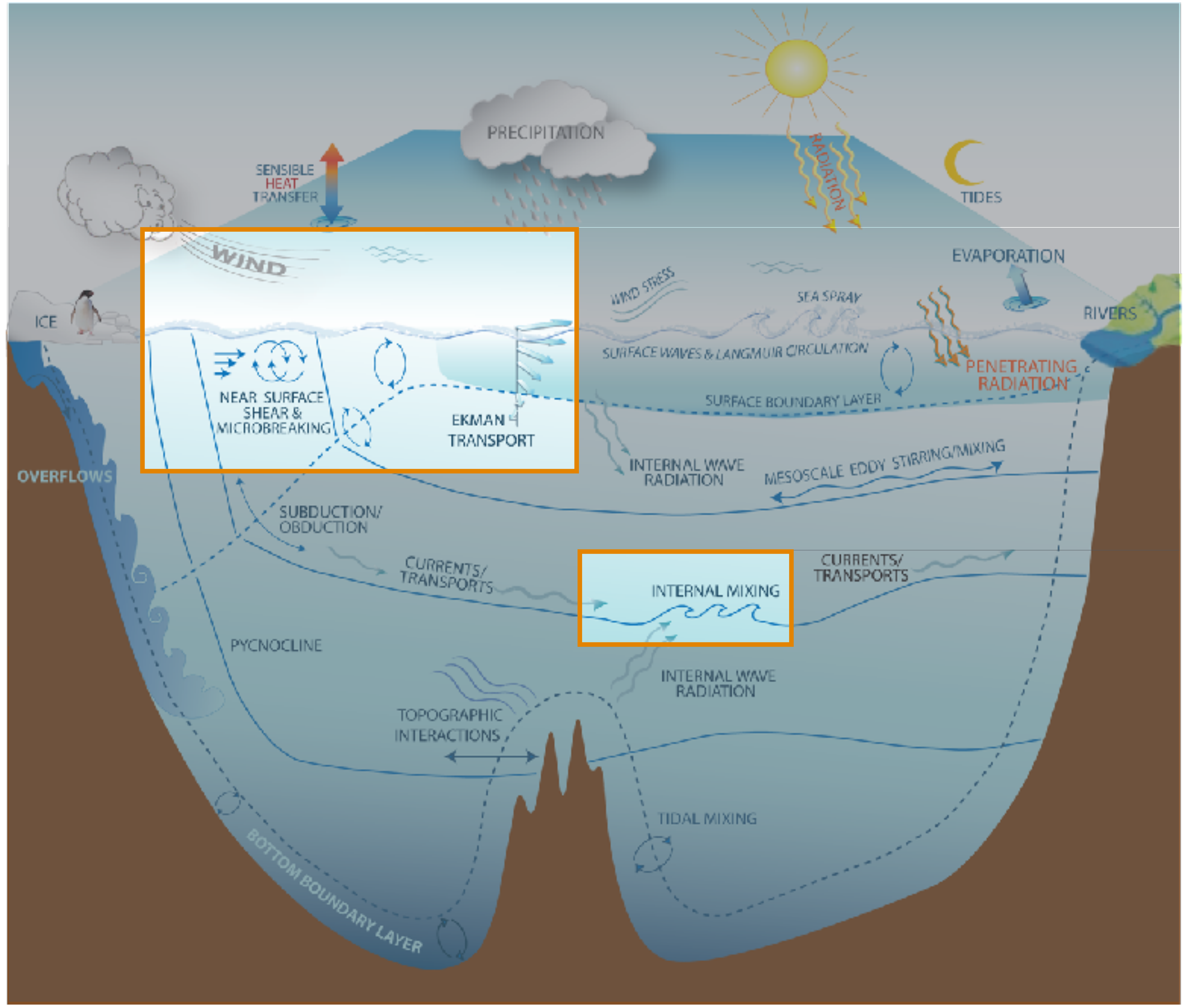


Figure from Von Storch and Zwiers, 1999

Forced Convection: Wind-driven circulation



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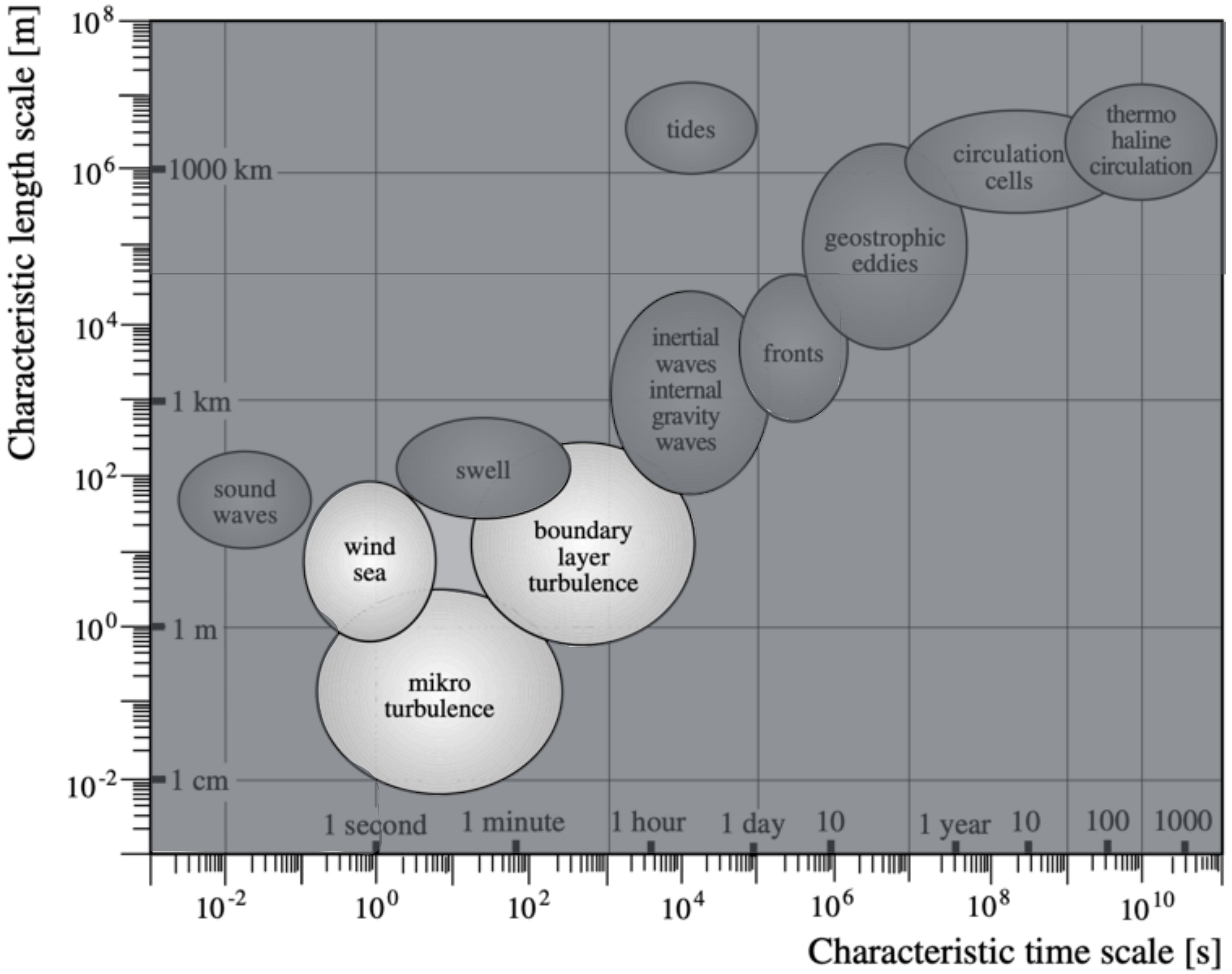

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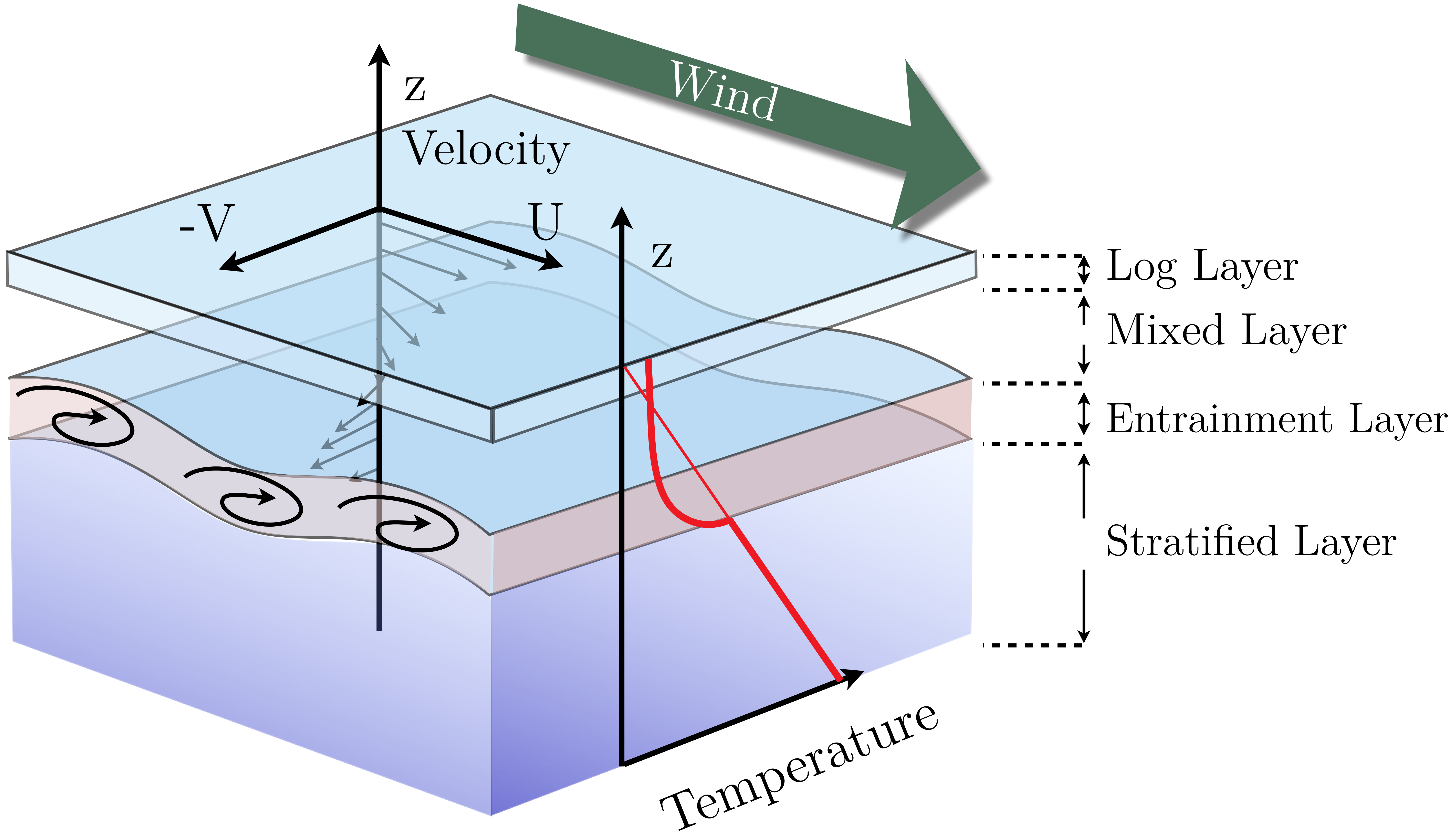
Wind-driven surface dynamics



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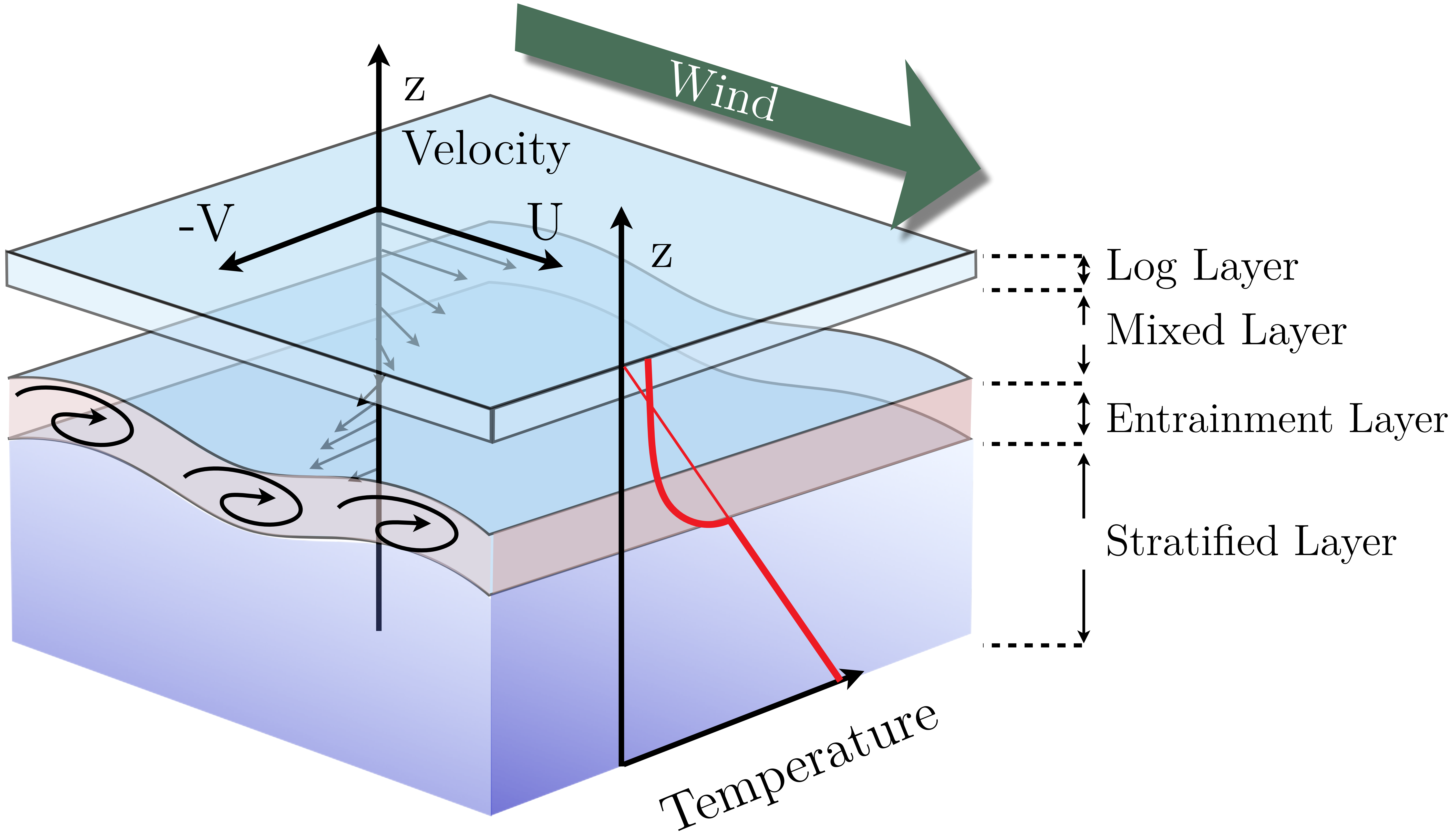
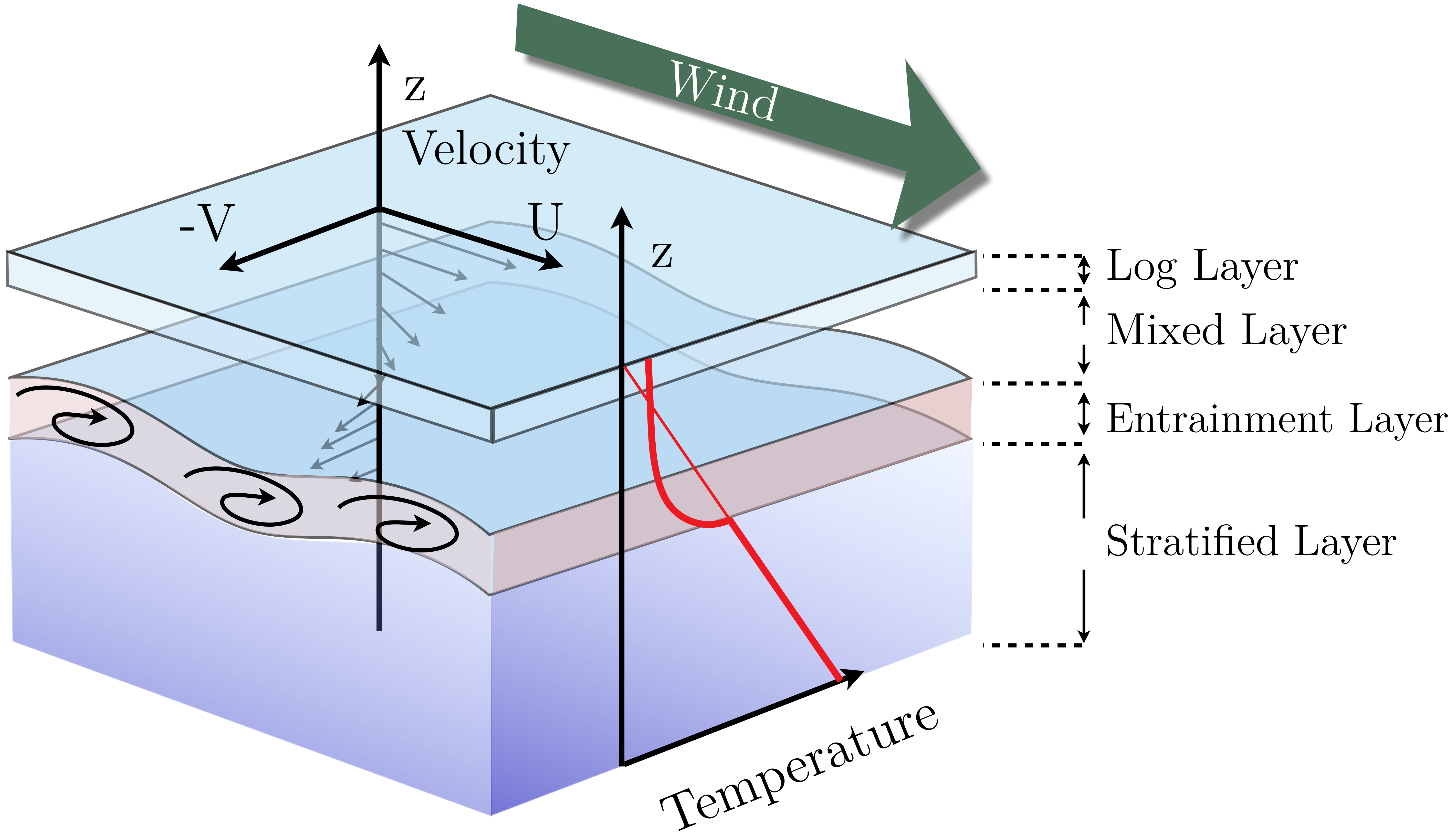


Surface friction

$$\tau = C_d(U_s - U_a)\langle \mathbf{U_s} - \mathbf{U_a} \rangle$$

$$= \overline{u_j' w'} + \nu \partial_z u_j = u_*^2$$

Wind-driven surface dynamics



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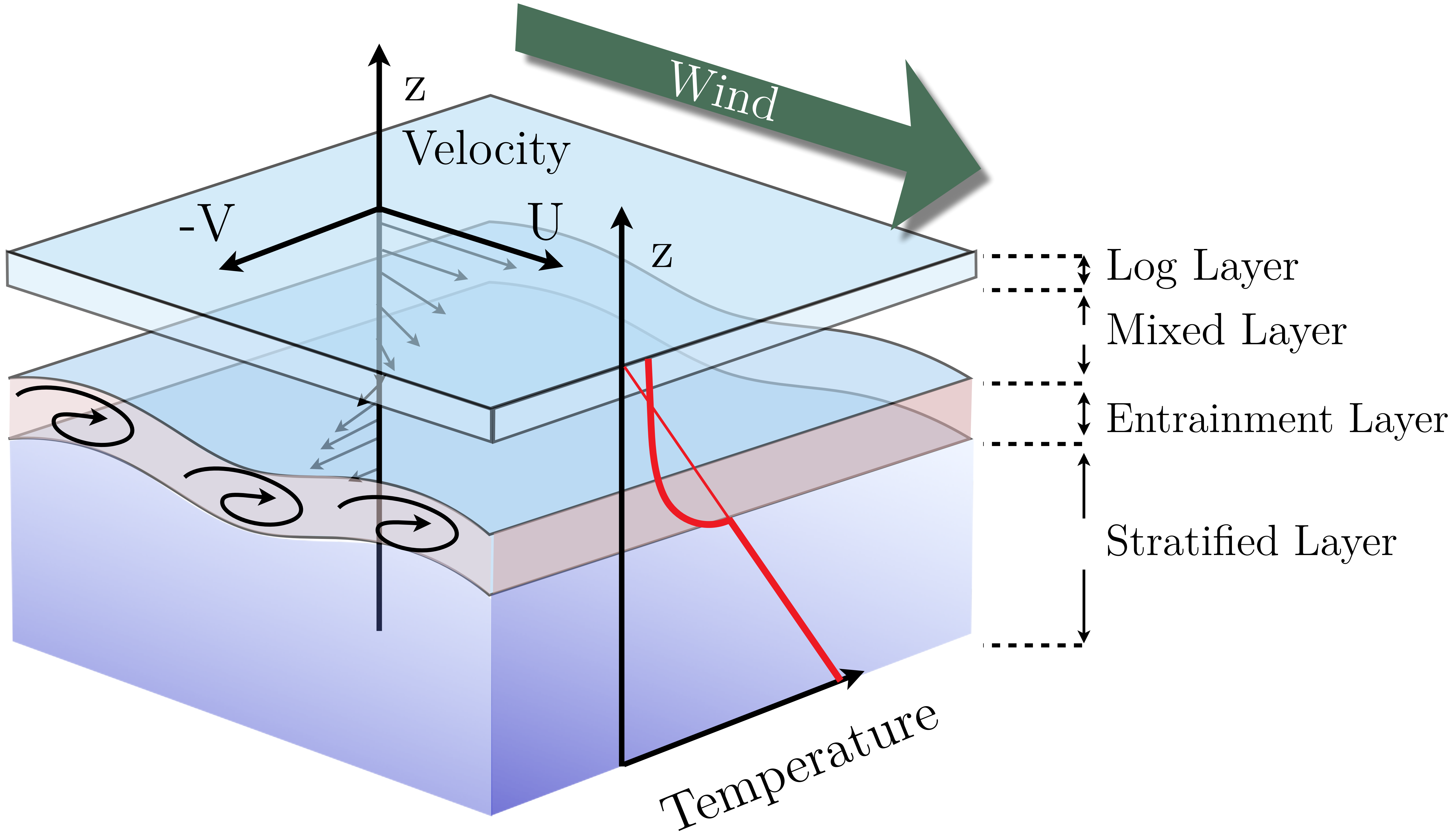
Mixed Layer growth $h(t) \propto t^{1/2}$

- Monin–Obukhov (1954)
- Kato & Phillips (1969)
- Pollard et al., (1973)

Wind-driven surface dynamics



Surface friction

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$$= \overline{u_j' w'} + \nu \partial_z u_j = u_*^2$$

In rotating frame*

Inertial oscillations triggered

Velocity profil → Ekman Spiral

Different deepening rate

Wind-driven surface dynamics

Long term deepening with rotation

- Isolate the inertial oscillation / Ekman-type velocity

Deepening rate in rotation $h(t) \propto t^{1/4}$

*Coppin, Deremble and Sommeria, submitted to GAFD, 2025

Wind-driven surface dynamics

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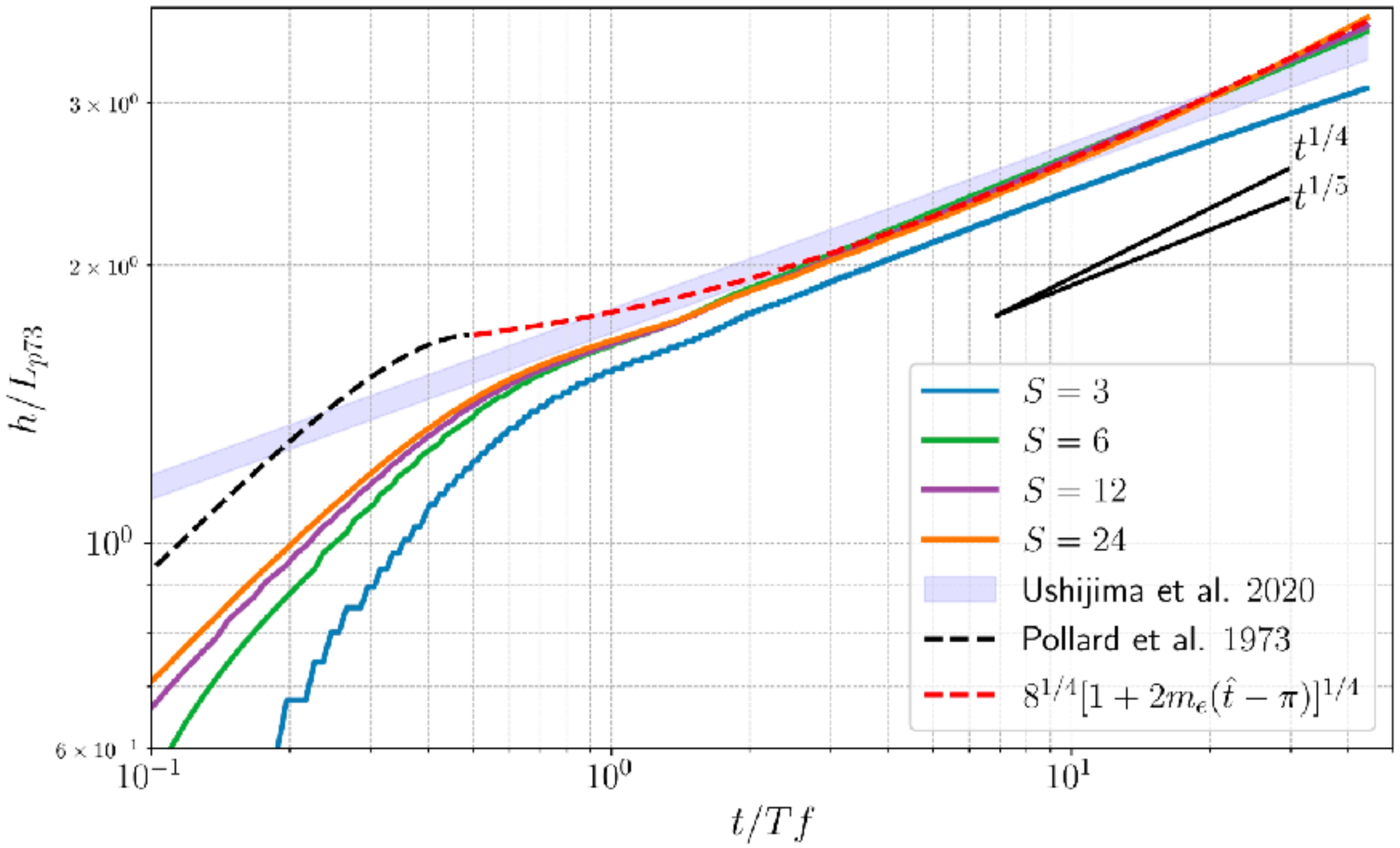
Confrontation to numerical model

1D **Turbulent model closure** $(k-\epsilon)$

Two prognostic equations:

- Turbulent kinetic energy
- Dissipation



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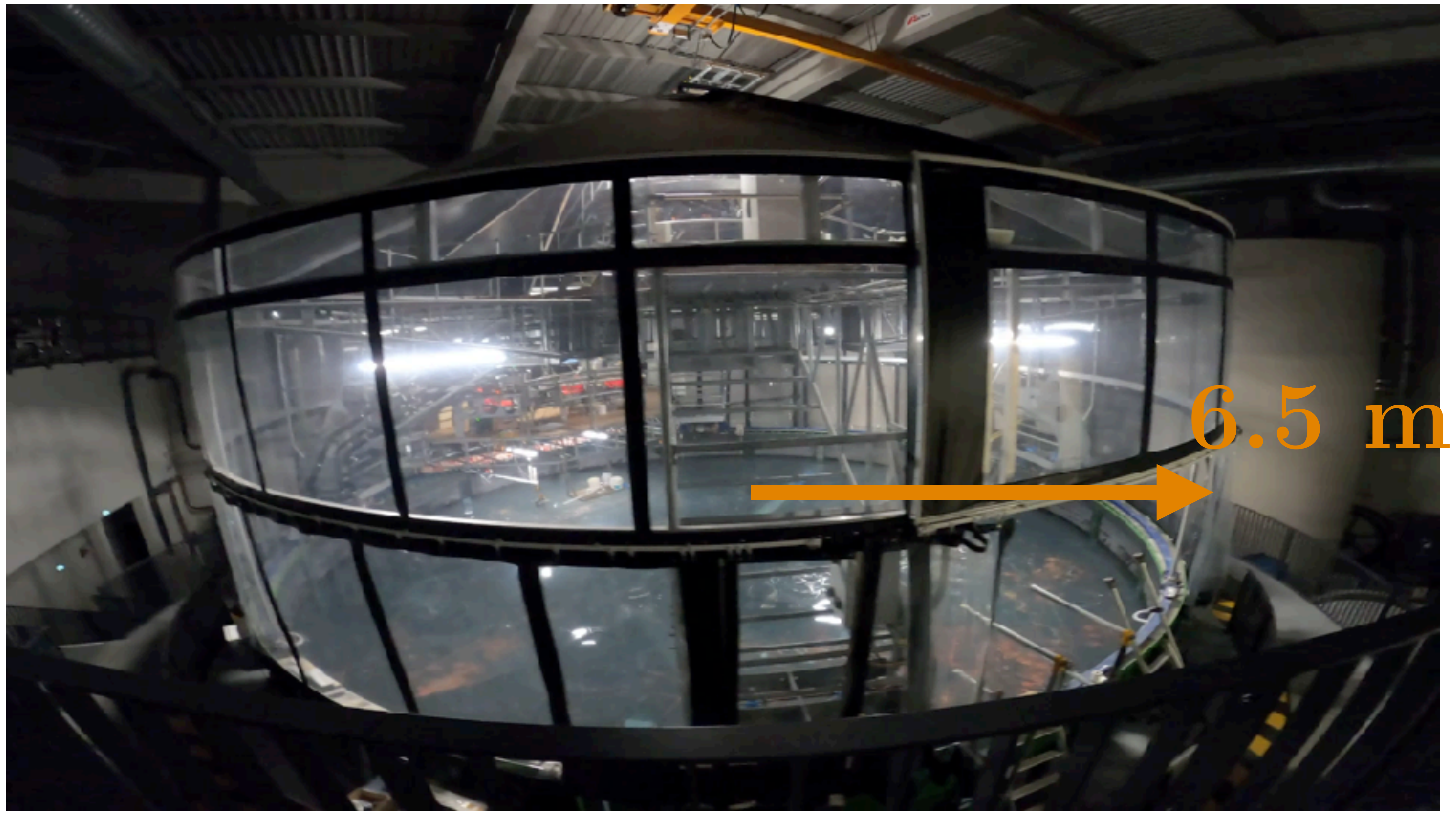

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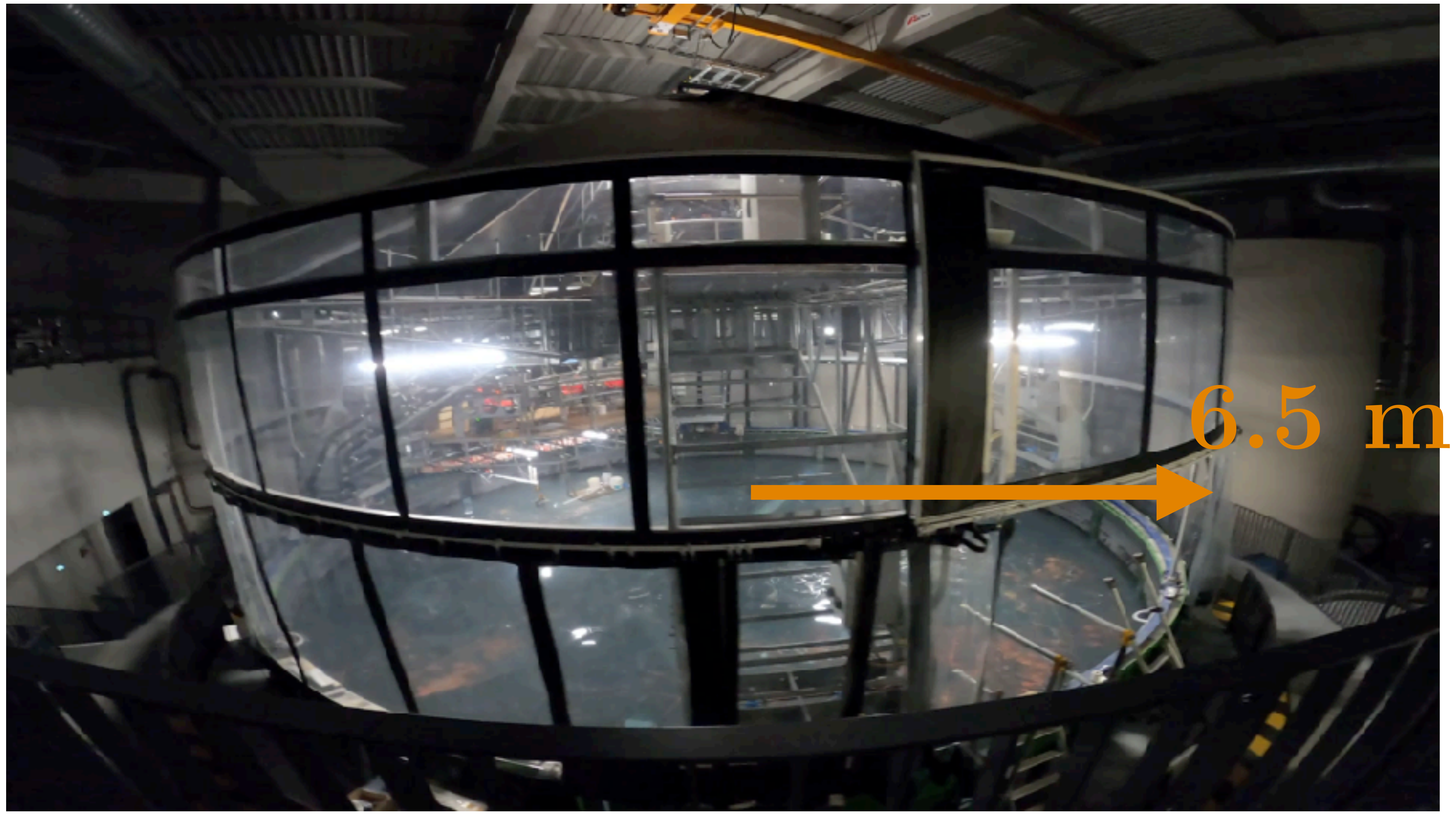
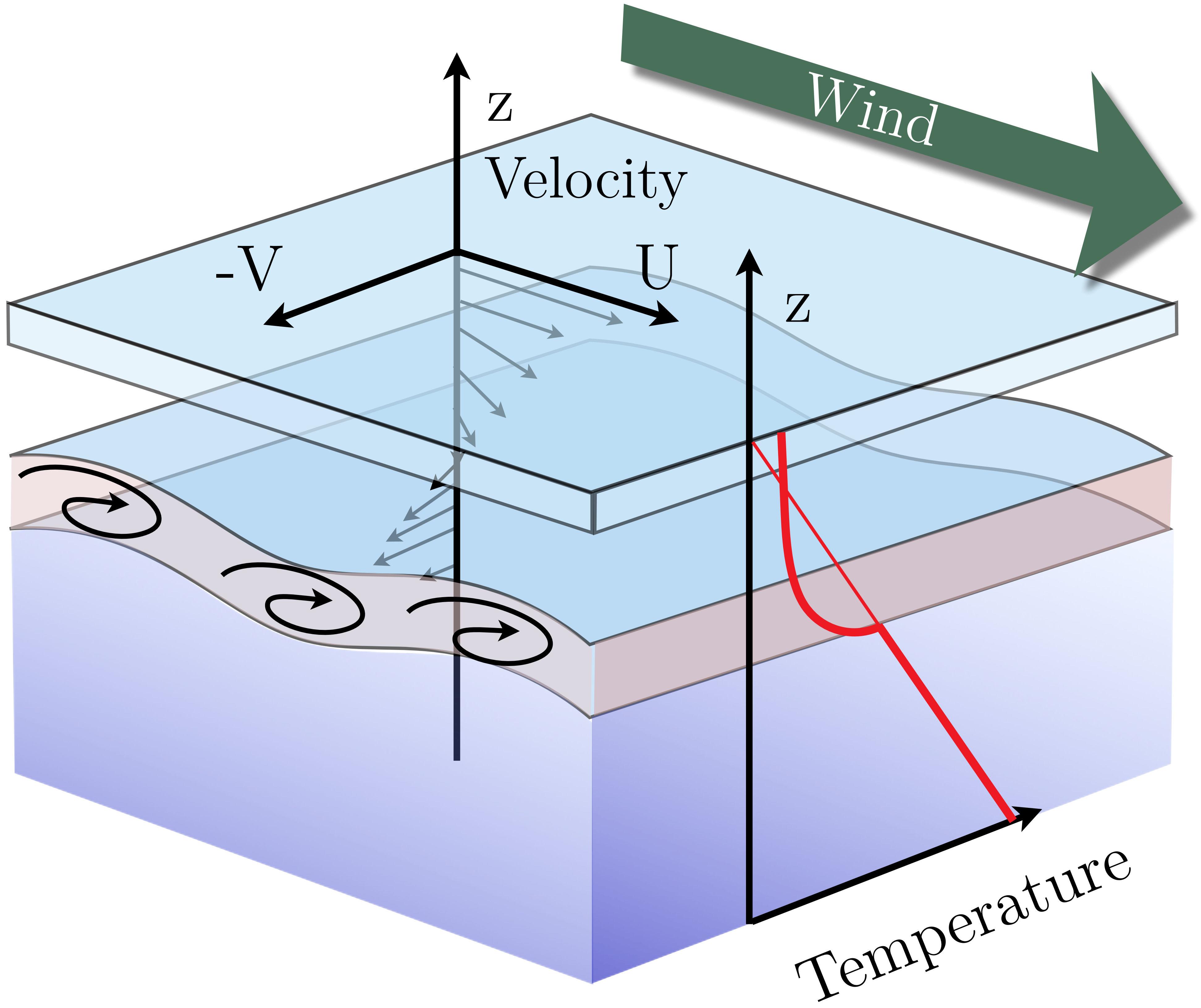


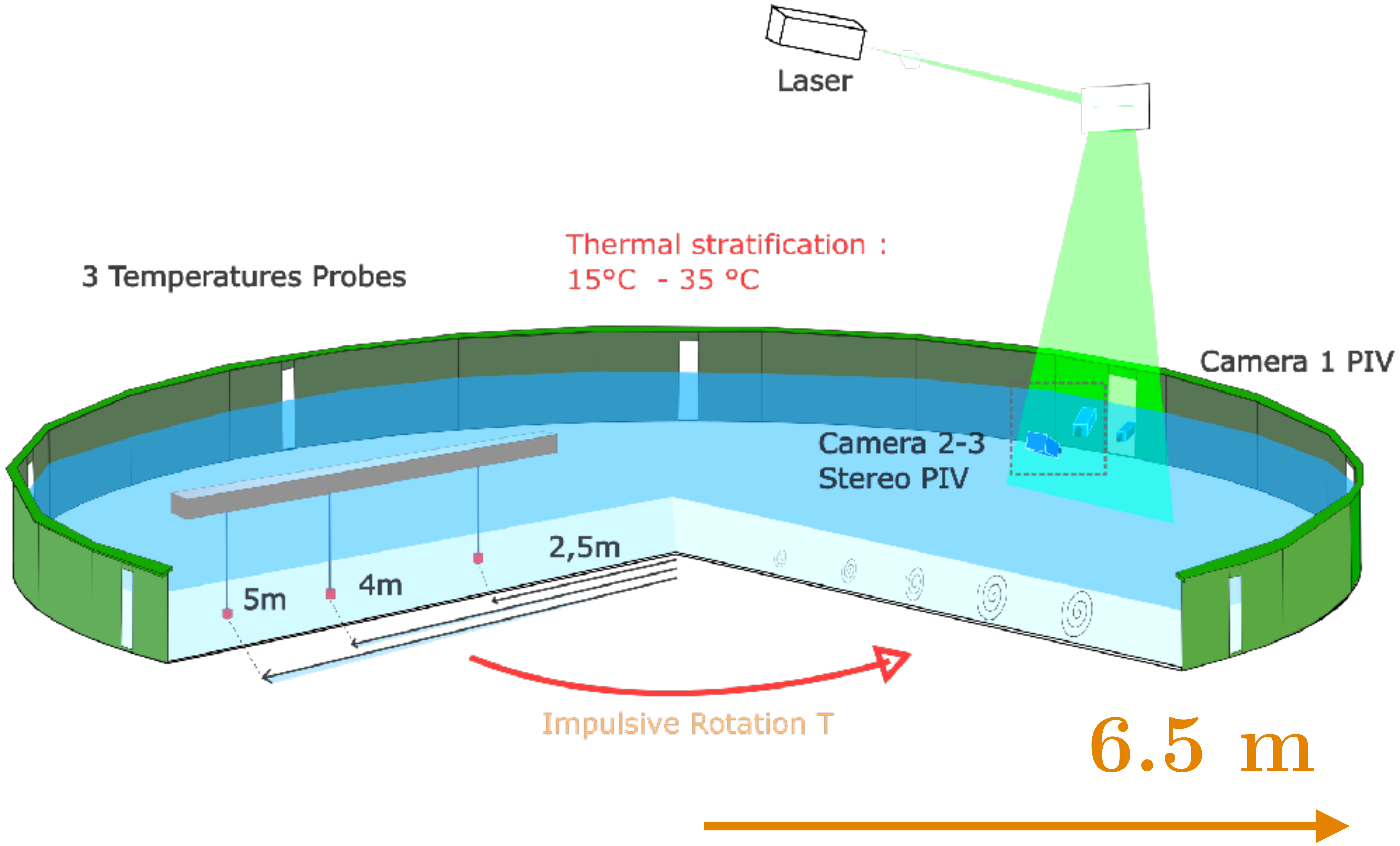
The aim is to experimentally support this development

1. Direct measurement of the mixed layer growth
2. Representation of the mean dynamics
3. Representation of the turbulence

*Coppin, Deremble and Sommeria, submitted to GAFD, 2025

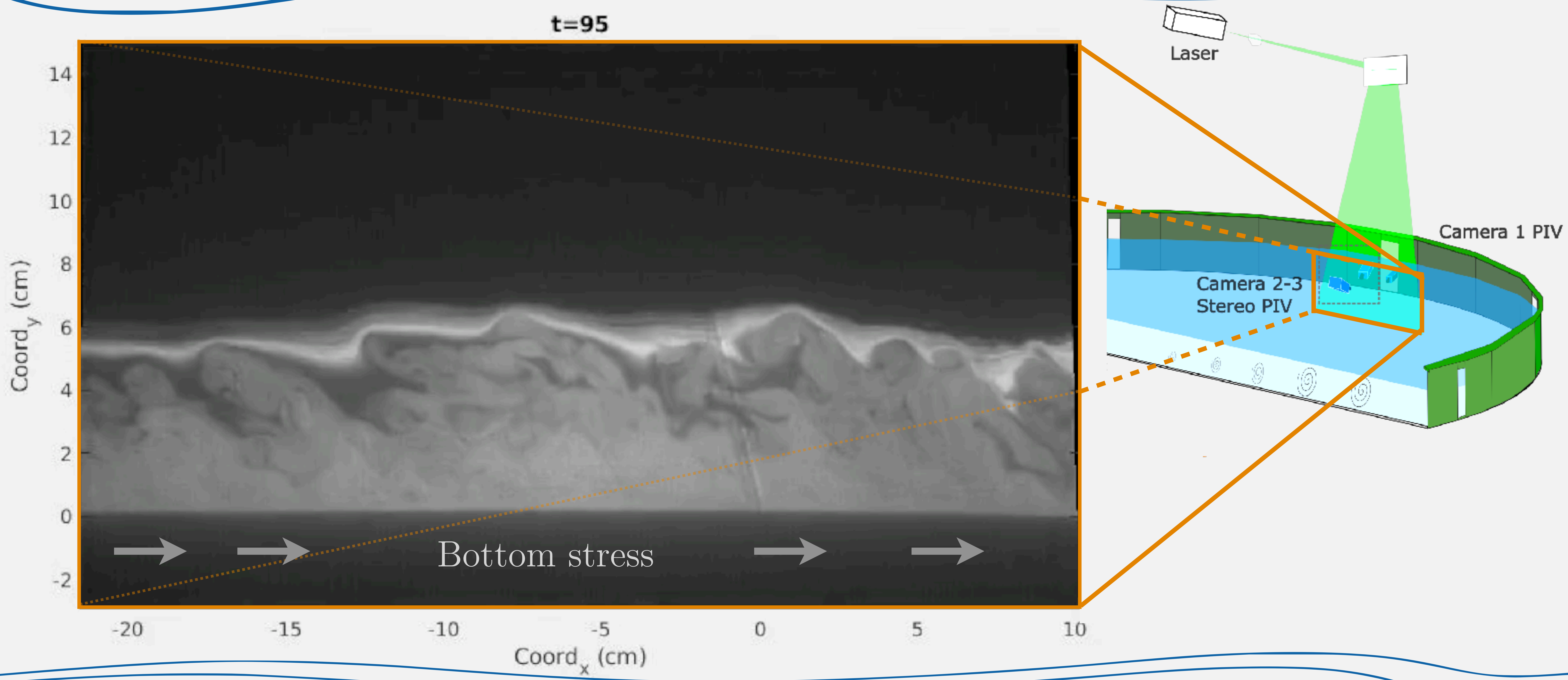
Experimental Ocean Mixed layer on Coriolis Plateform



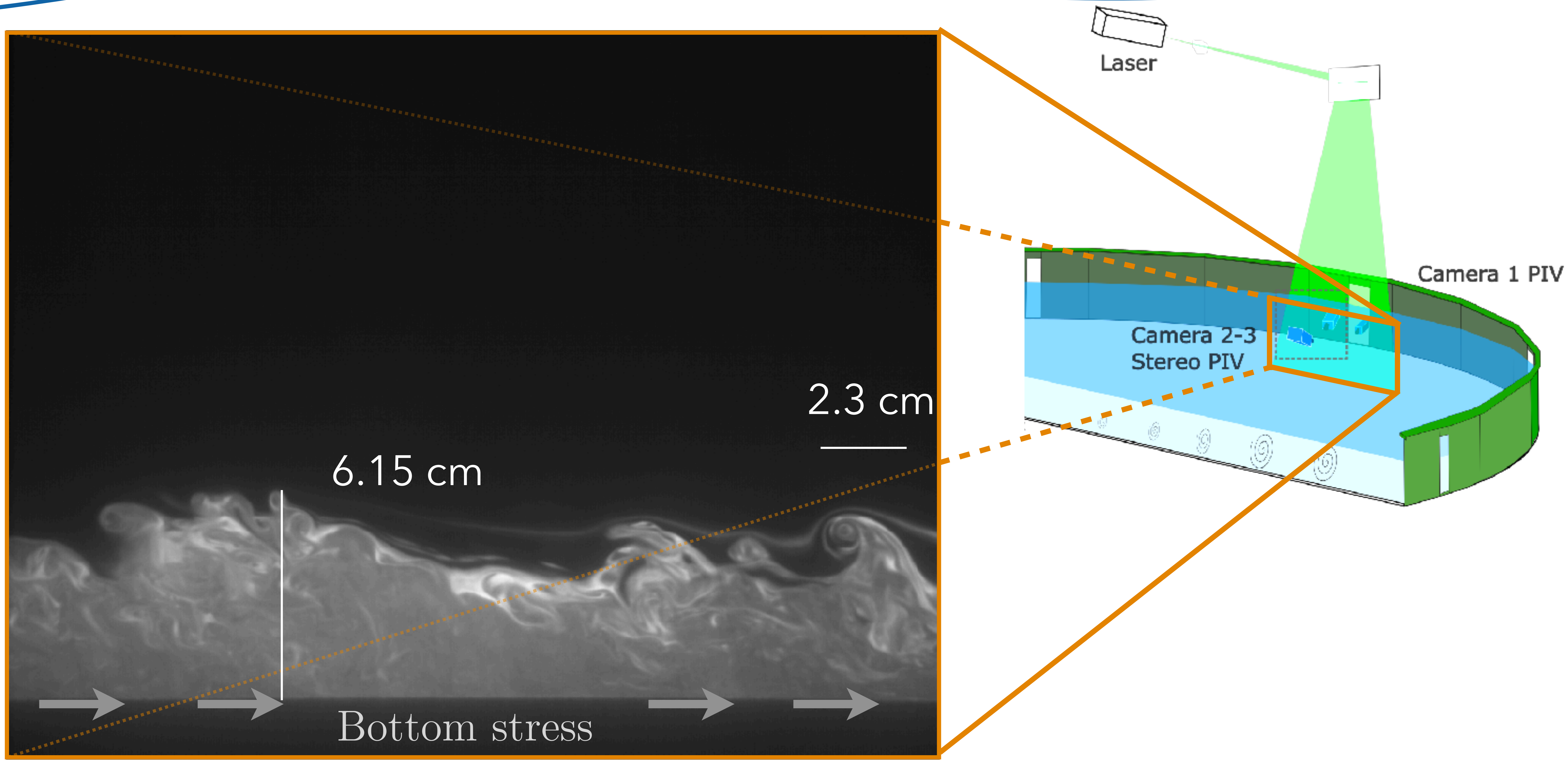


Upside down configuration

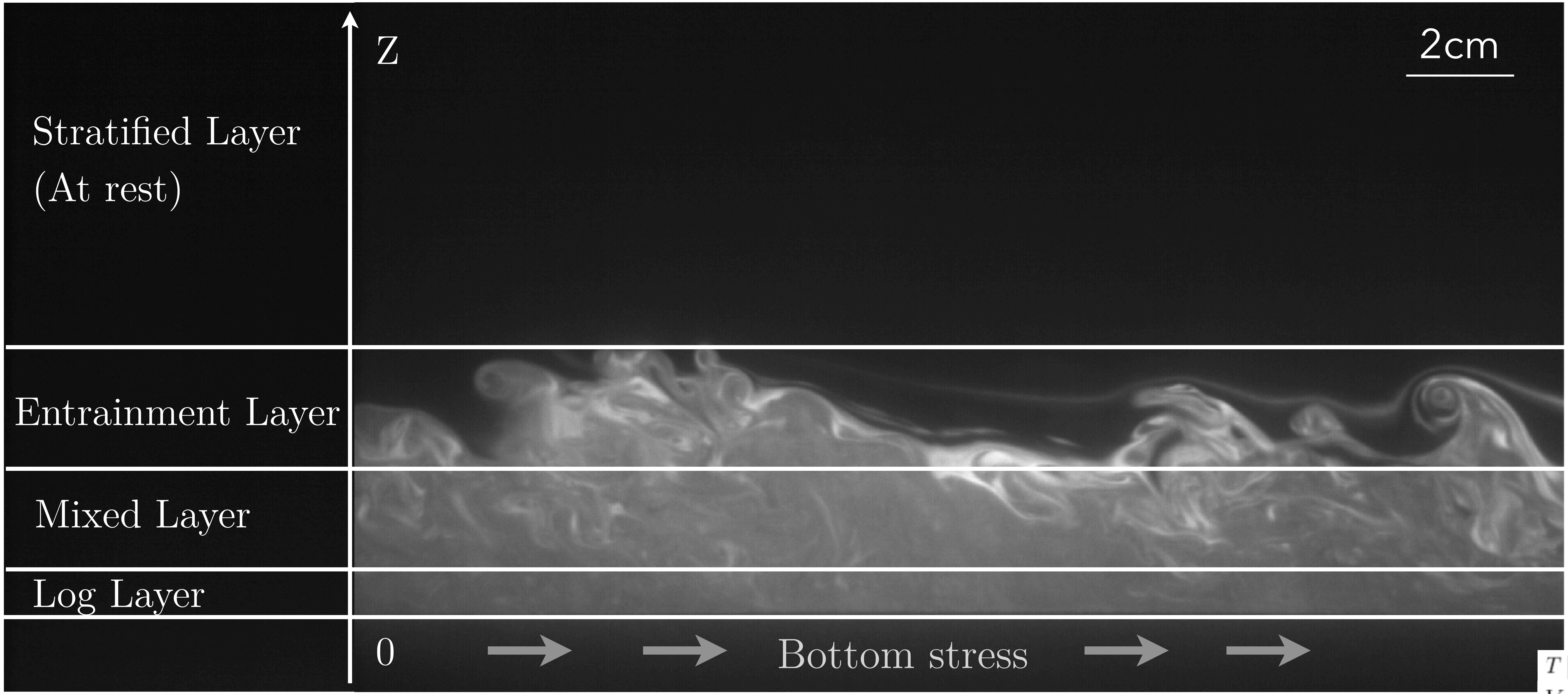
Experimental Ocean Mixed layer on Coriolis Plateform

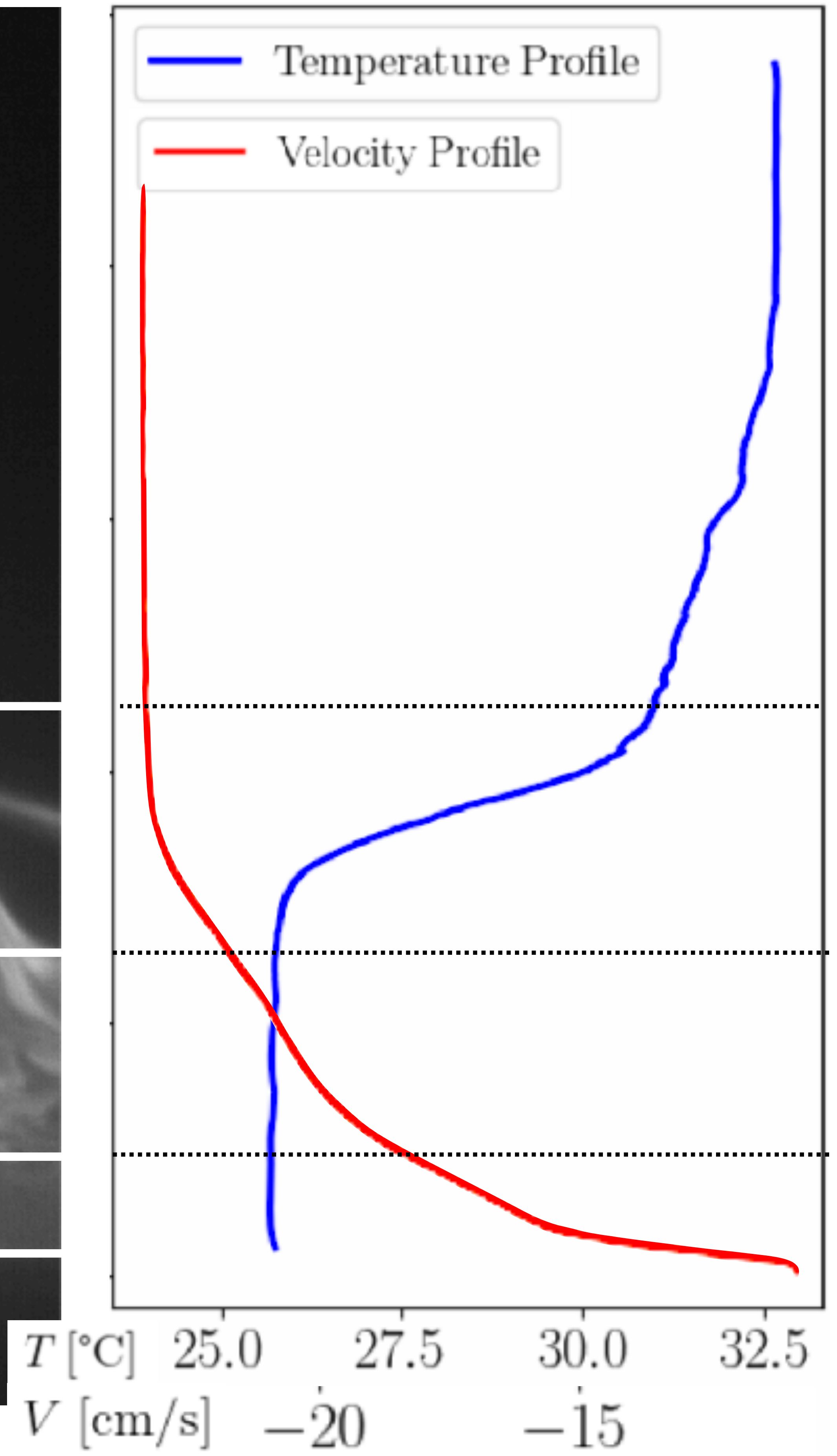


Experimental Ocean Mixed layer on Coriolis Plateform



Vertical Structure of Entrainment





Vertical Structure of Entrainment

Vertical Structure of Entrainment

Erosion of the stratification

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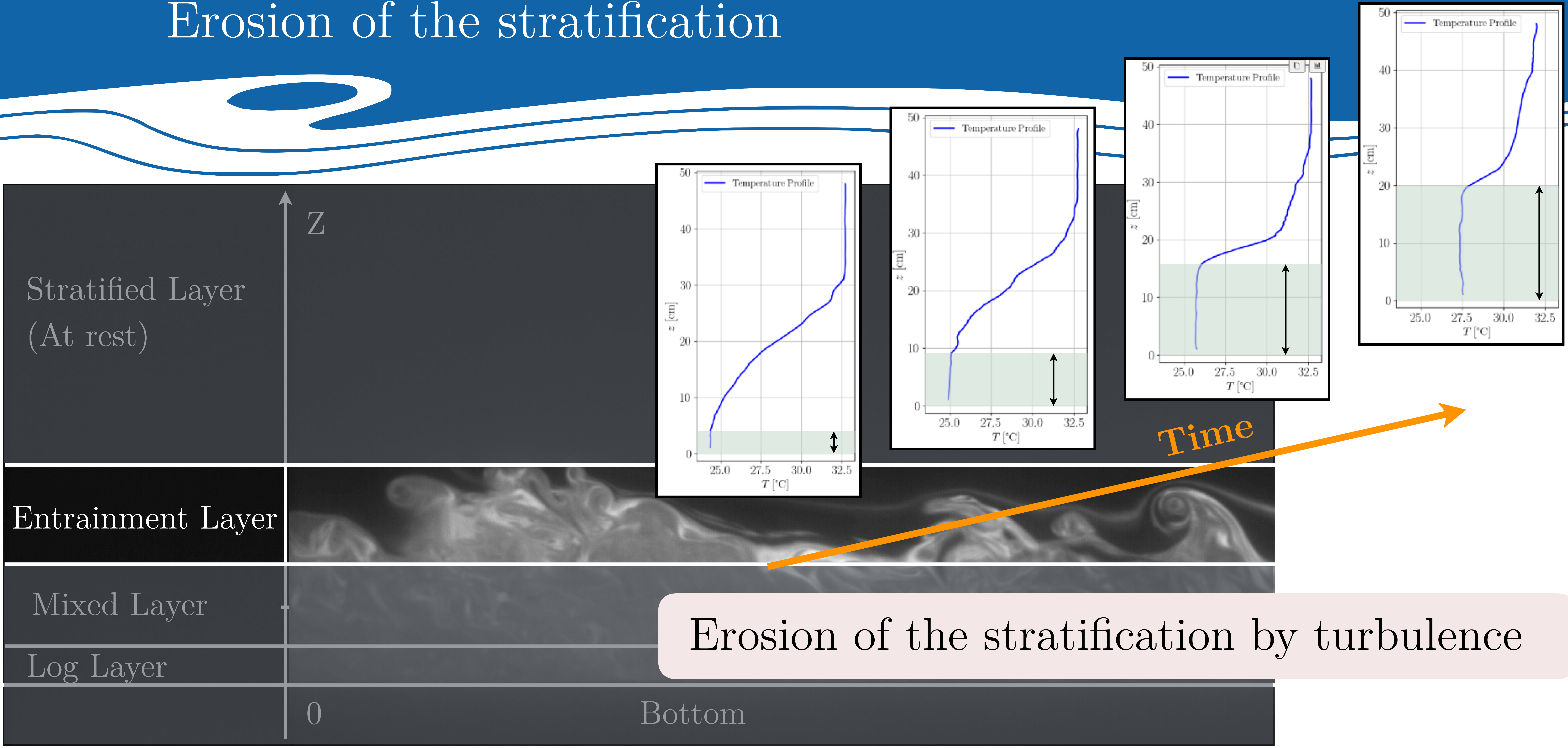


Erosion of the stratification by turbulence

Direct measurement of the Mixed Layer deepening

Velocity profile

Temperature profile

Tracer Mixing

Direct measurement of the Mixed Layer deepening

Velocity profile

Temperature profile

Tracer Mixing

Observation constraints

- Camera cover only 14 first cm

Direct measurement of the Mixed Layer deepening

Velocity profile | **Temperature profile** | **Tracer Mixing**

Observation constraints

- Camera cover only 14 first cm

Spin-up constraints

- Conservation of the angular momentum (diffusion in the interior)

Direct measurement of the Mixed Layer deepening

Velocity profile | **Temperature profile** | **Tracer Mixing**

Observation constraints

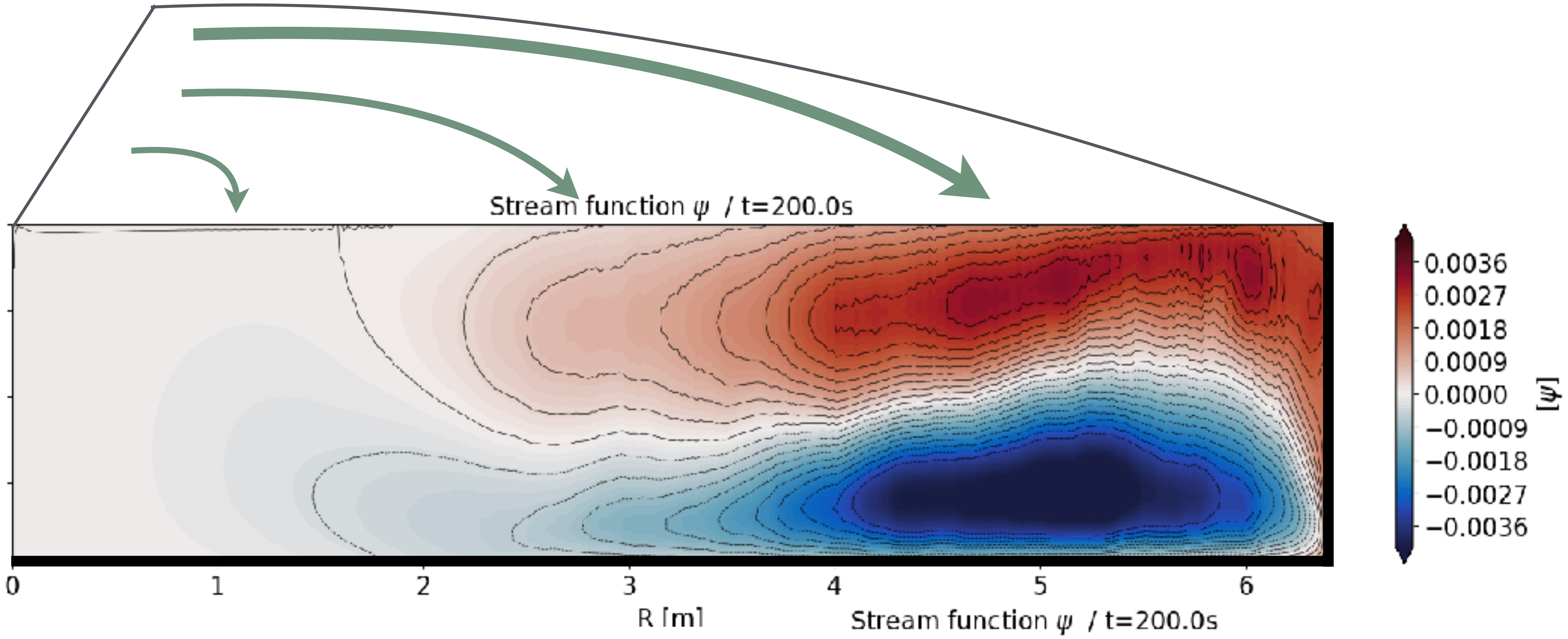
- Camera cover only 14 first cm

Spin-up constraints

- Conservation of the angular momentum (diffusion in the interior)

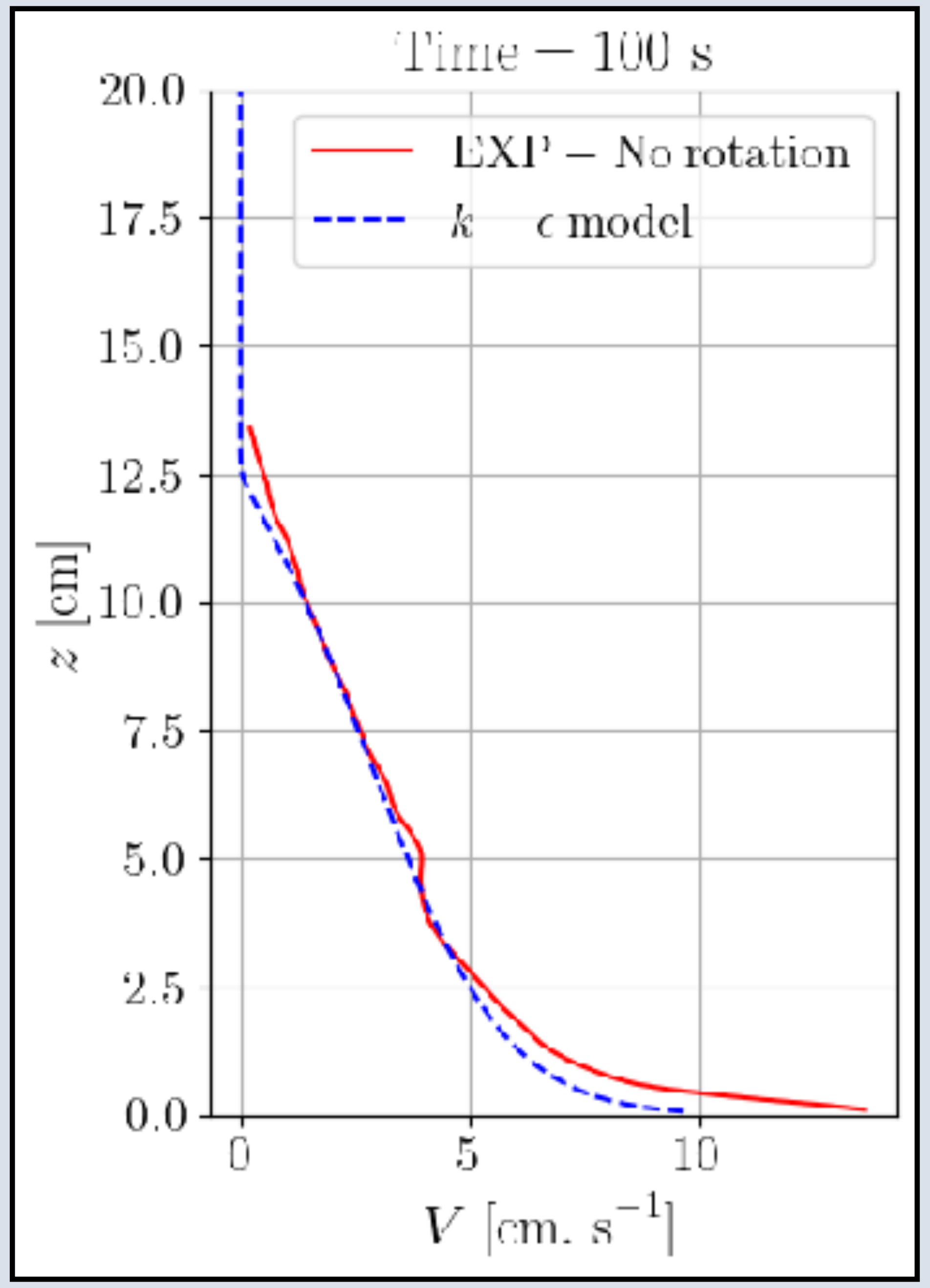
Geometrical constraints

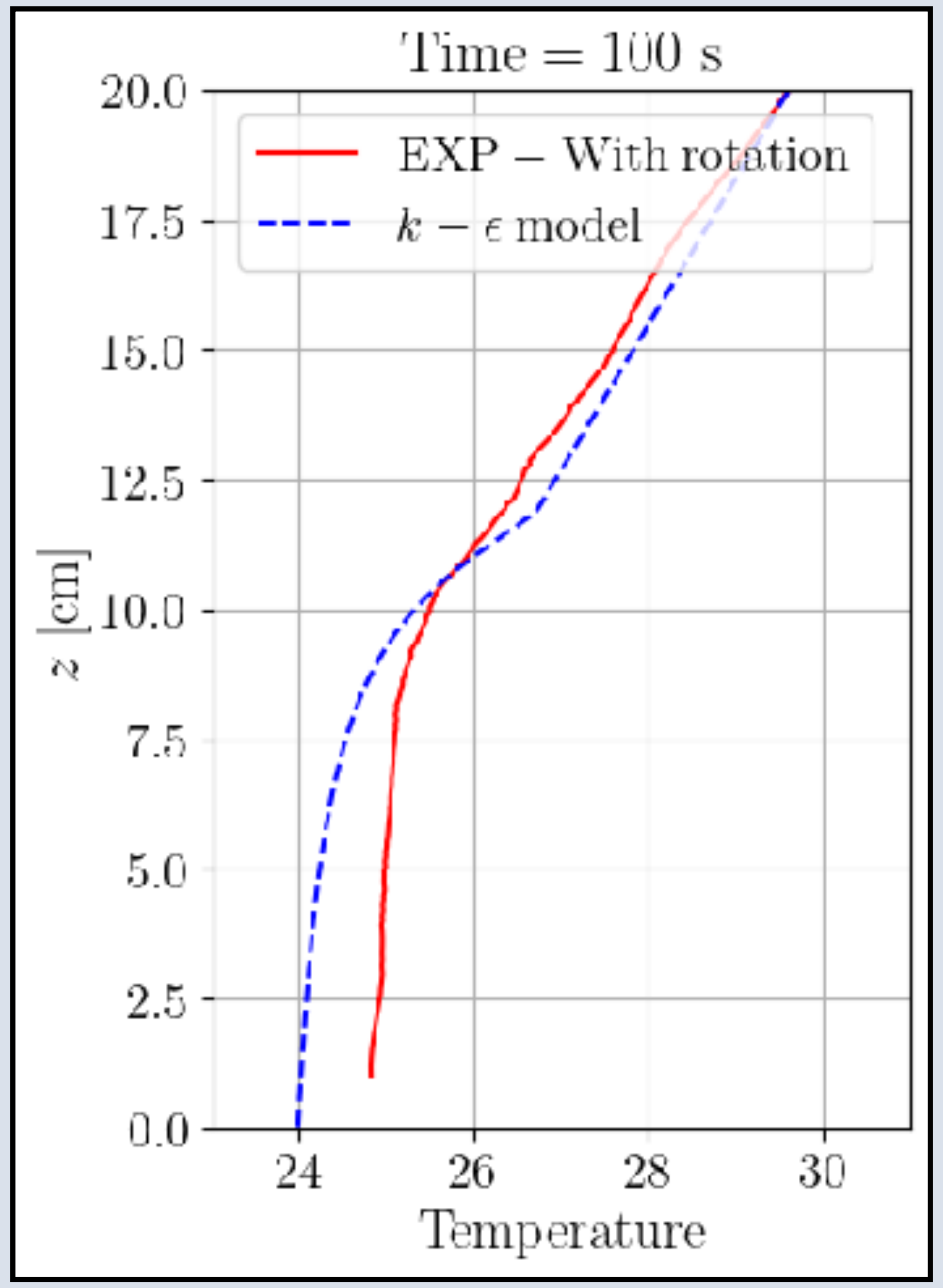
- Secondary circulation triggered
- Blocking of the inertial Oscillations



Validation of the $k - \epsilon$ representation of Ekman layer

Without Rotation





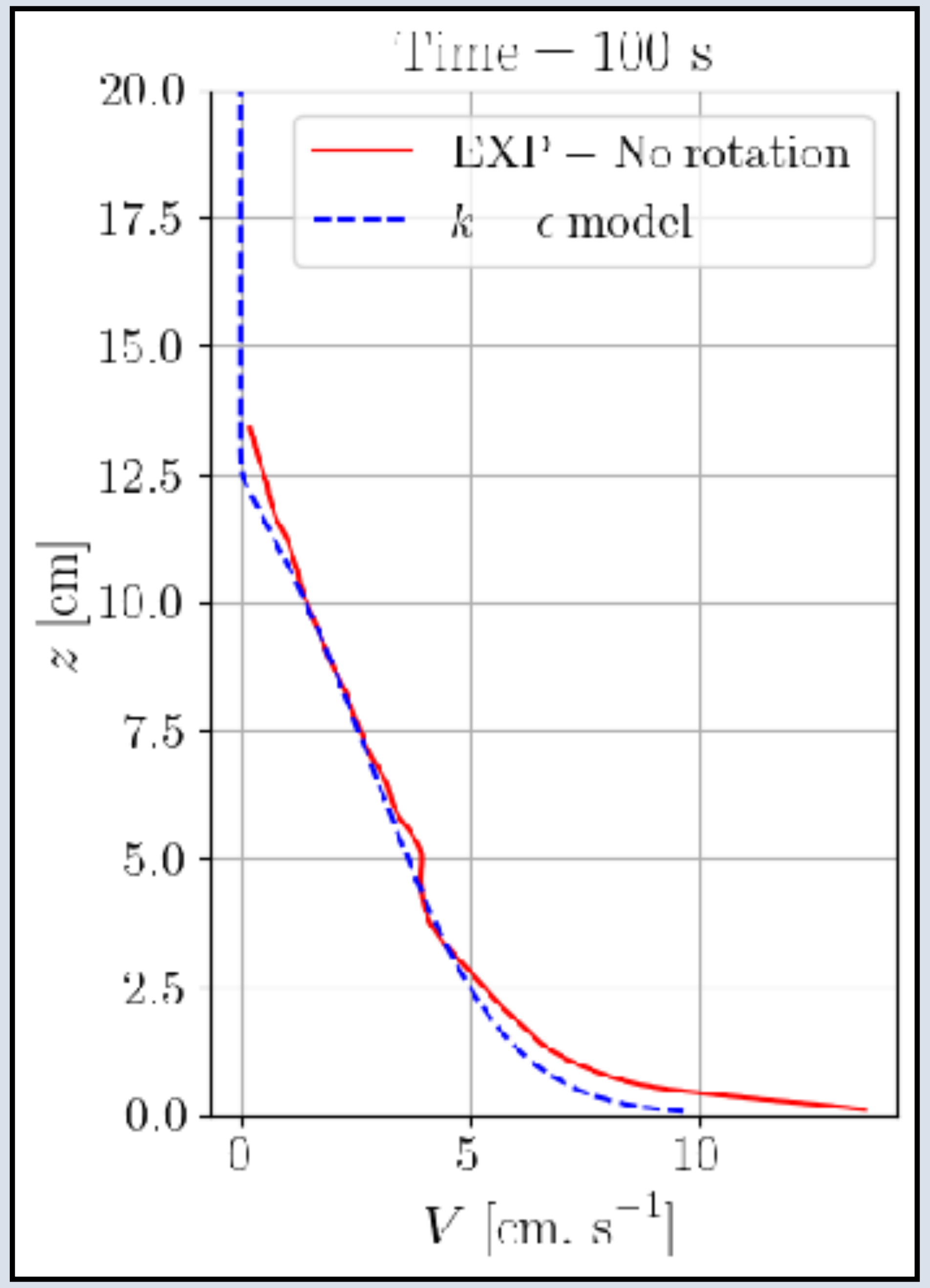
Azimuthal velocity

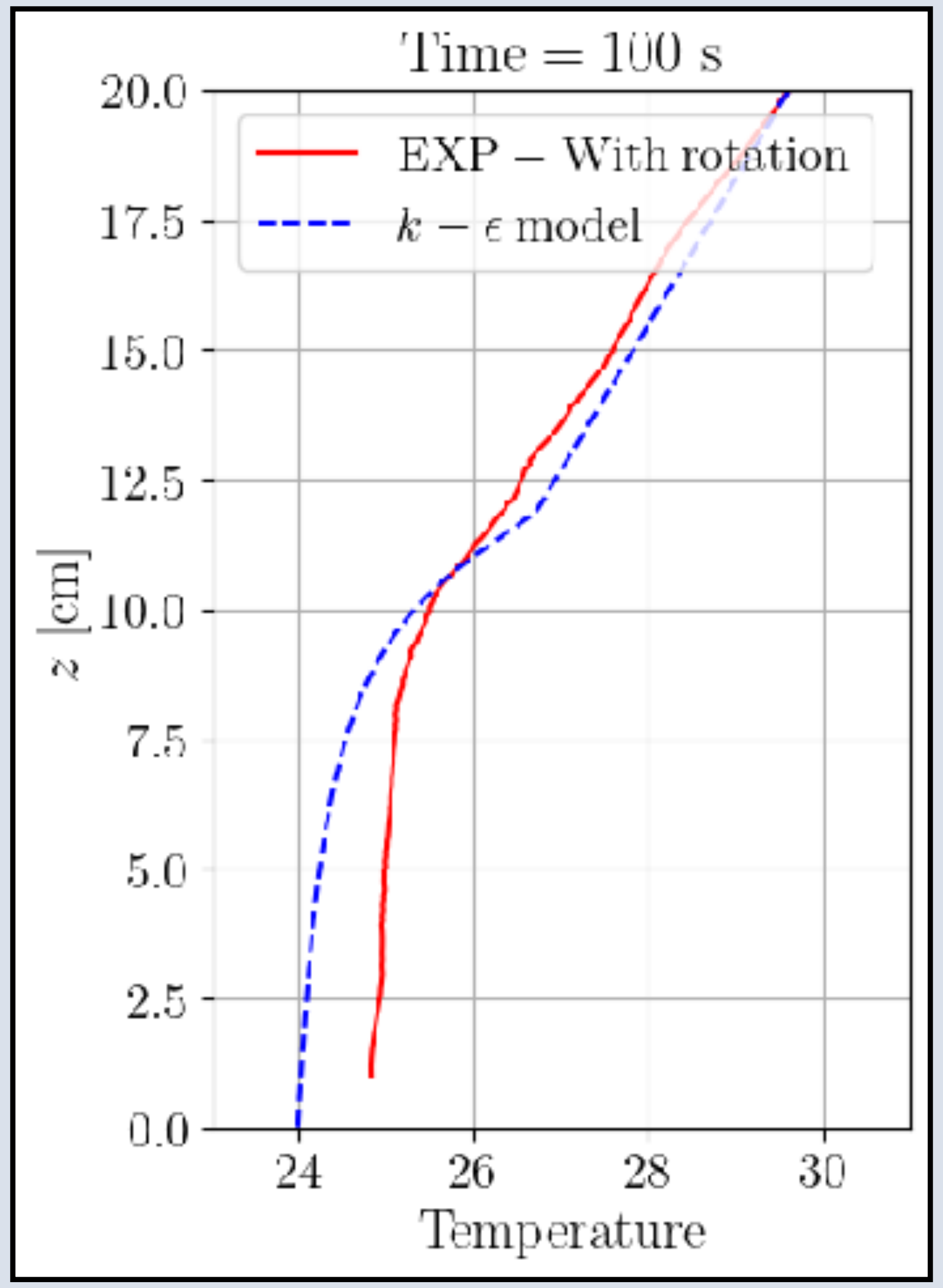
Density

*Note that the velocity and the thermal measurement are not at the same radius

Validation of the $k-\epsilon$ representation of Ekman layer

Without Rotation





Azimuthal velocity

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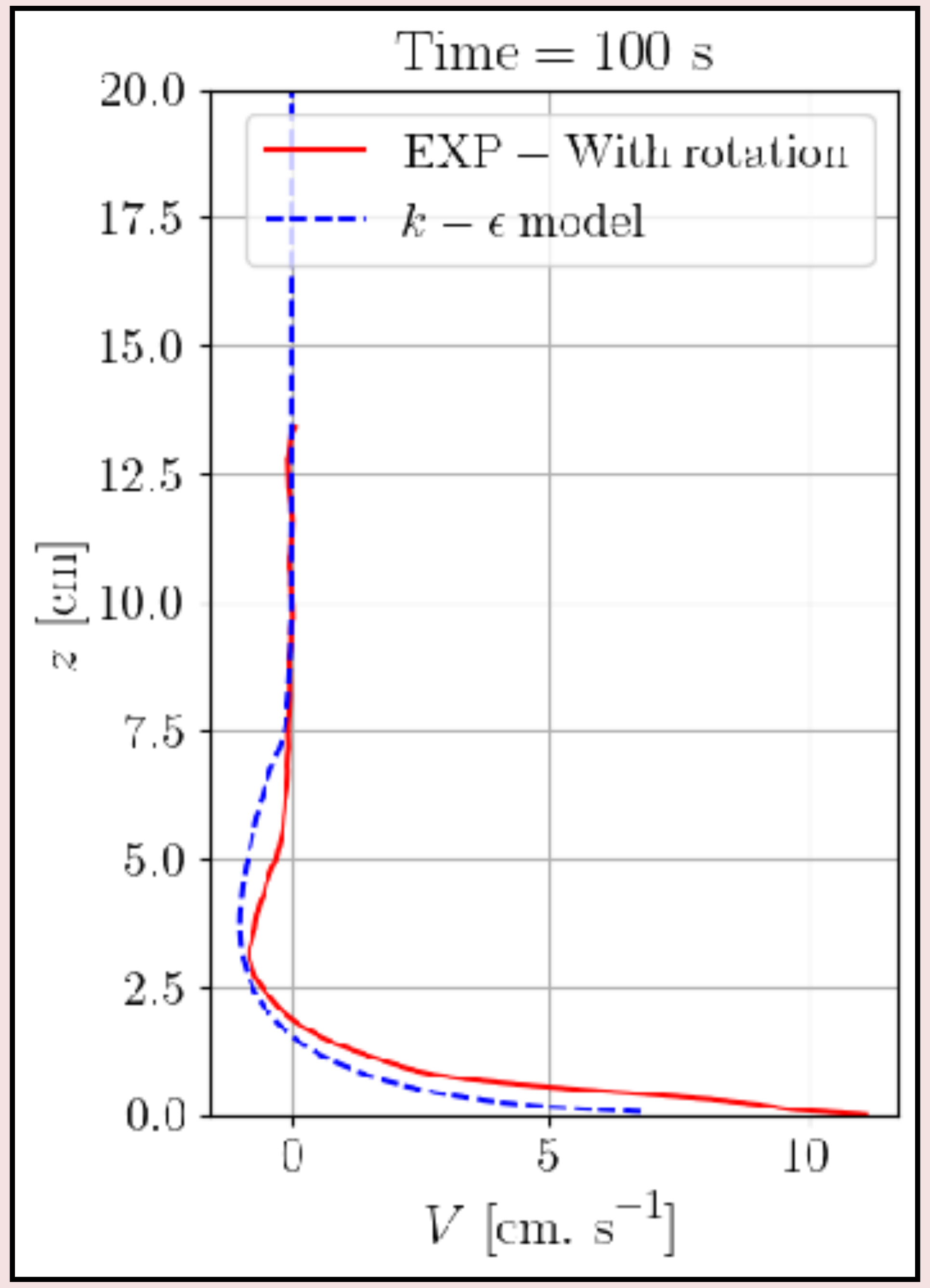
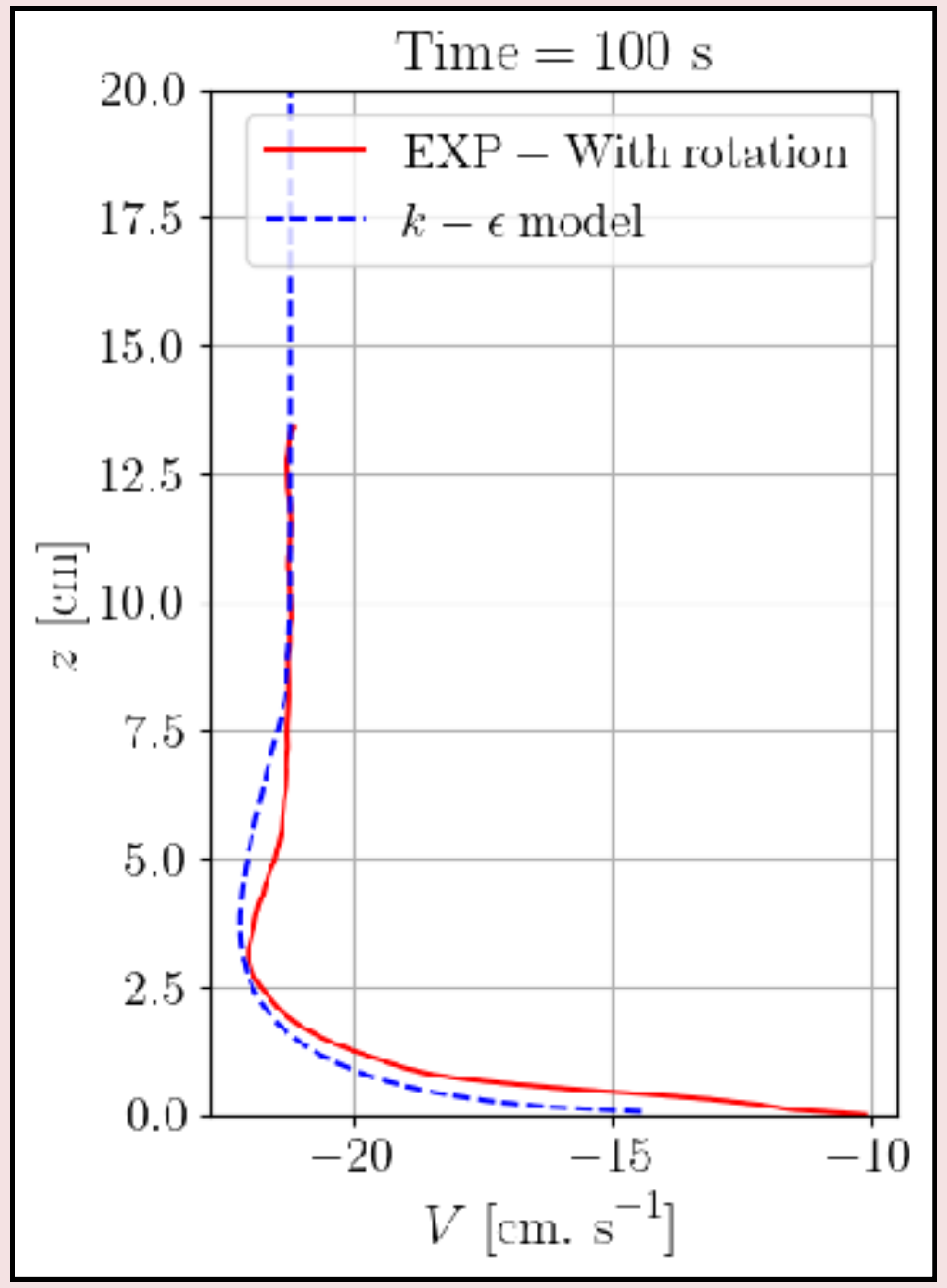
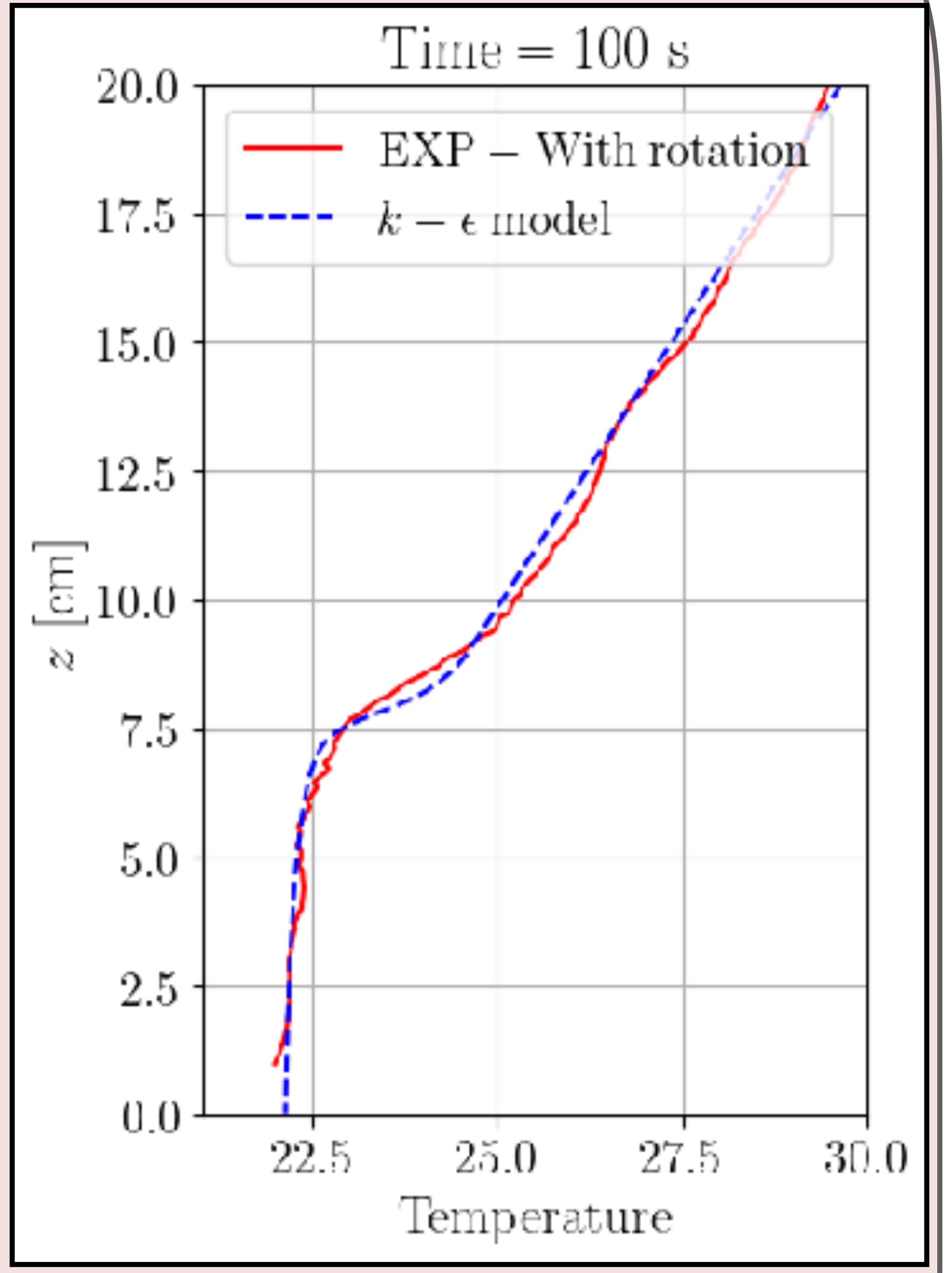
Azimuthal velocity



Radial velocity



Density

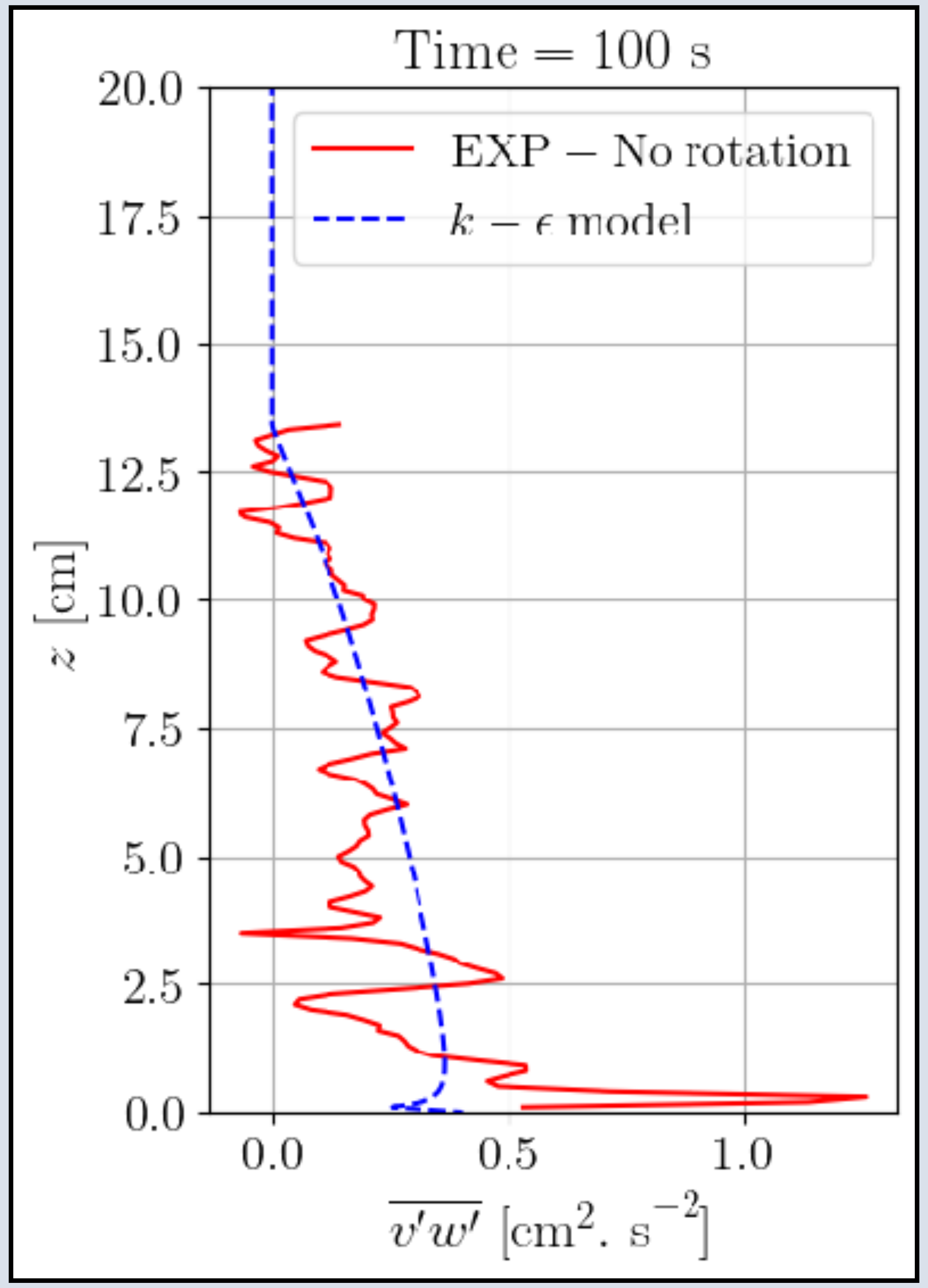


With Rotation

*Note that the velocity and the thermal measurement are not at the same radius

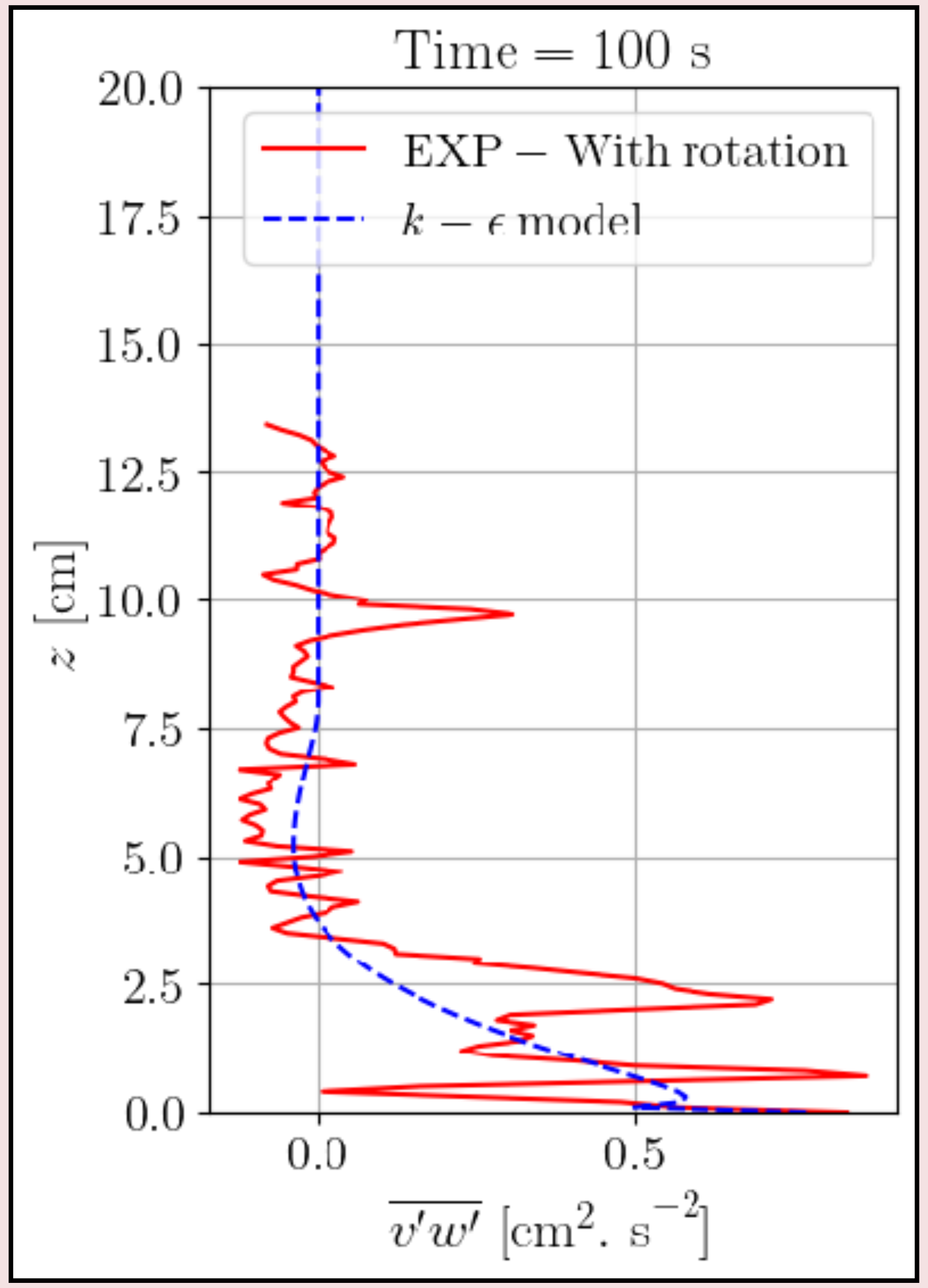
Validation of the $k-\epsilon$ representation of the turbulent flux

No Rotation

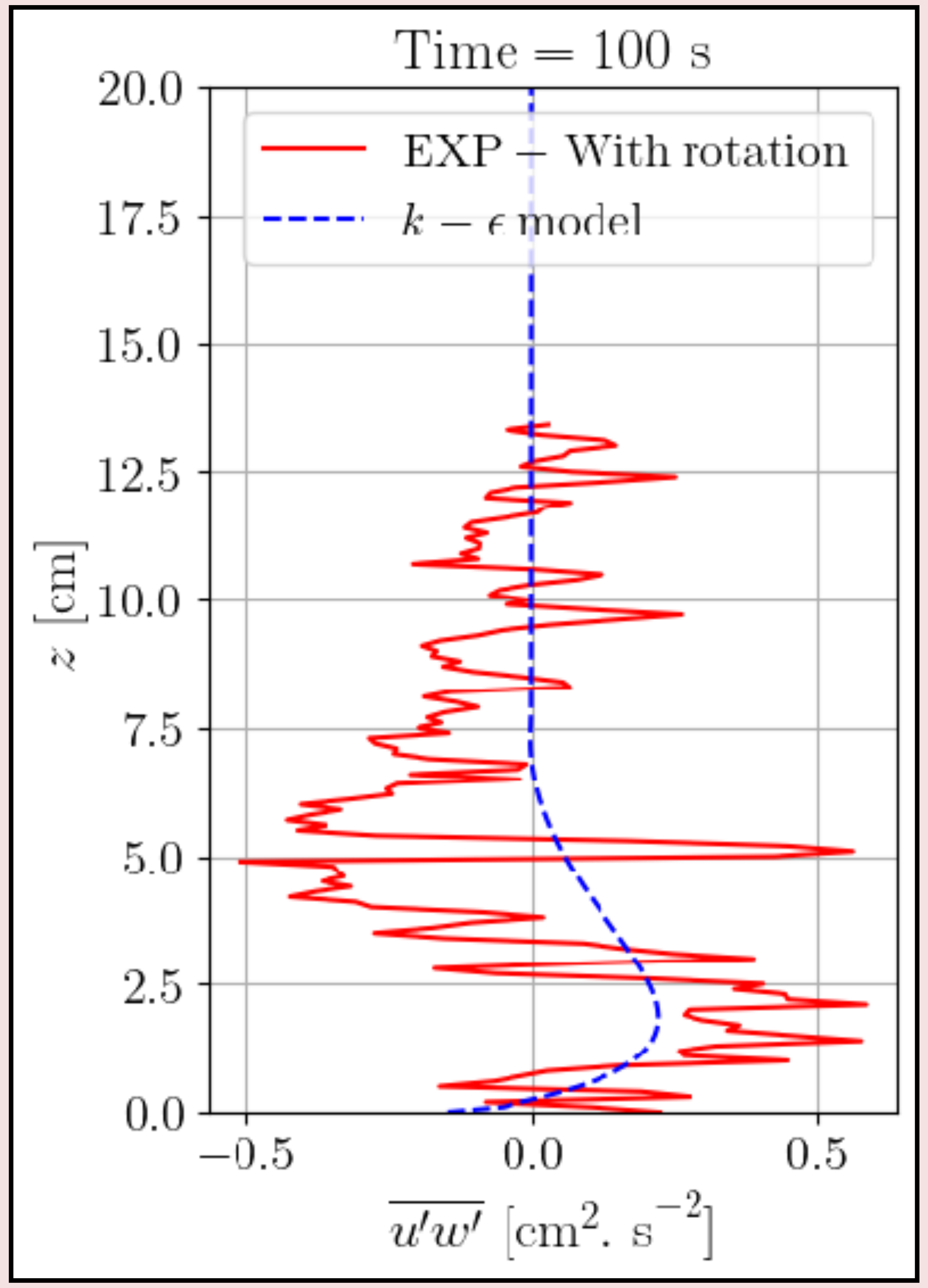


Azimuthal Turb flux

With Rotation



Azimuthal Turb flux



Radial Turb flux

Take -Home Messages

Validation of the $k - \epsilon$ turbulent model closure for frictionally driven entrainment

- The mean flow is correctly simulated
- The shear mechanisms are well represented in case of an turbulent Ekman case
- The erosion of the density is consistent with the experiment

Experimental validation of our developpement

- **Growth of the Wind driven mixed layer at long time at a rate $h(t) \propto t^{1/4}$**

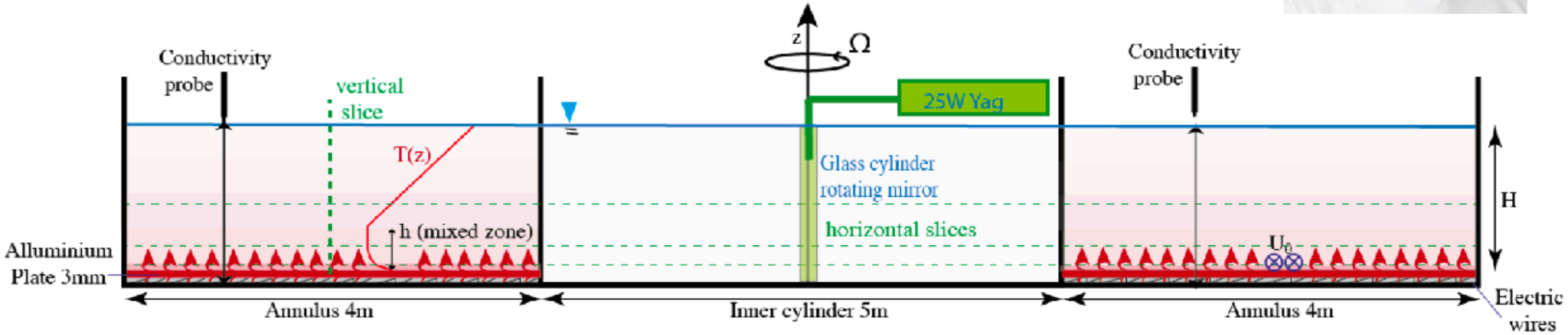
Future Works

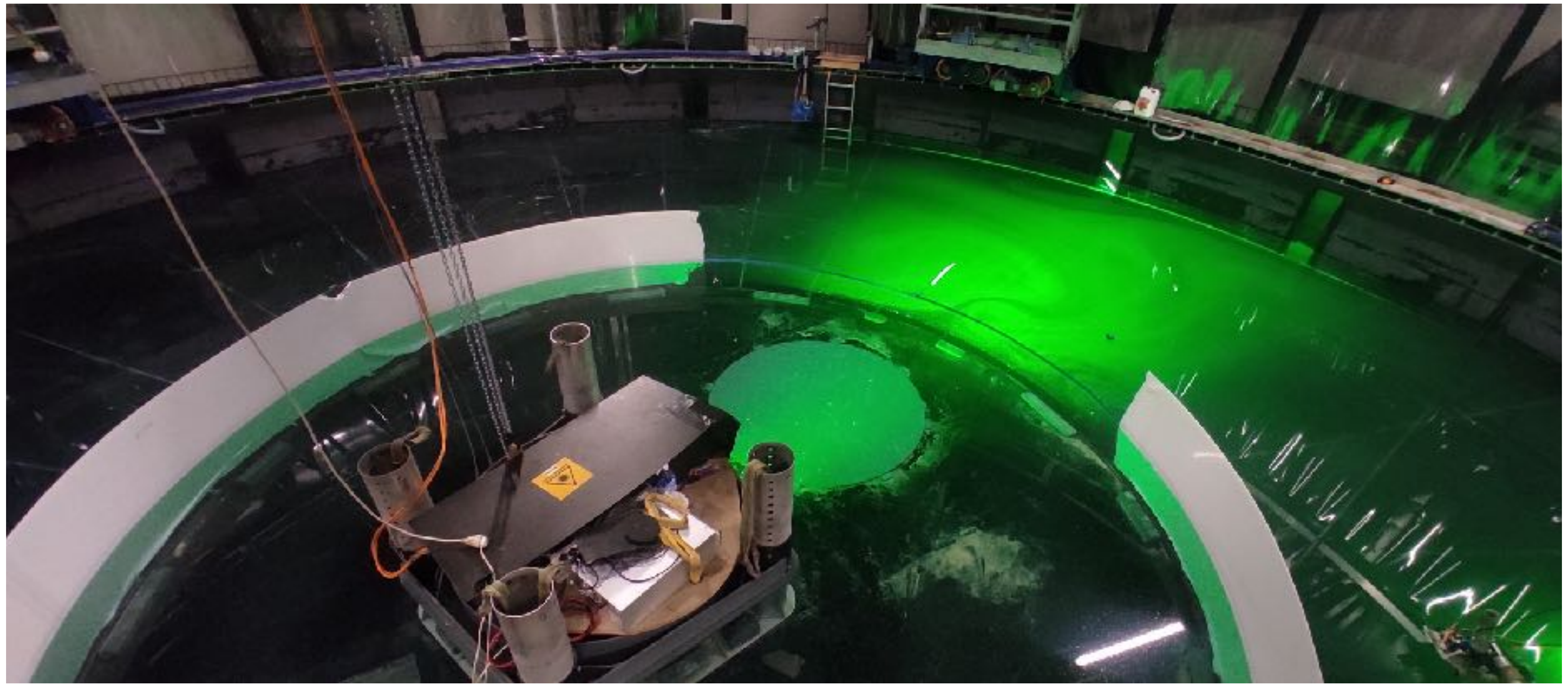
- Experimental Data base to test other turbulent moments closure
- New experimental campaign (Pr effect (salt) - higher stratification)

Next step : Free / Mixed Convection

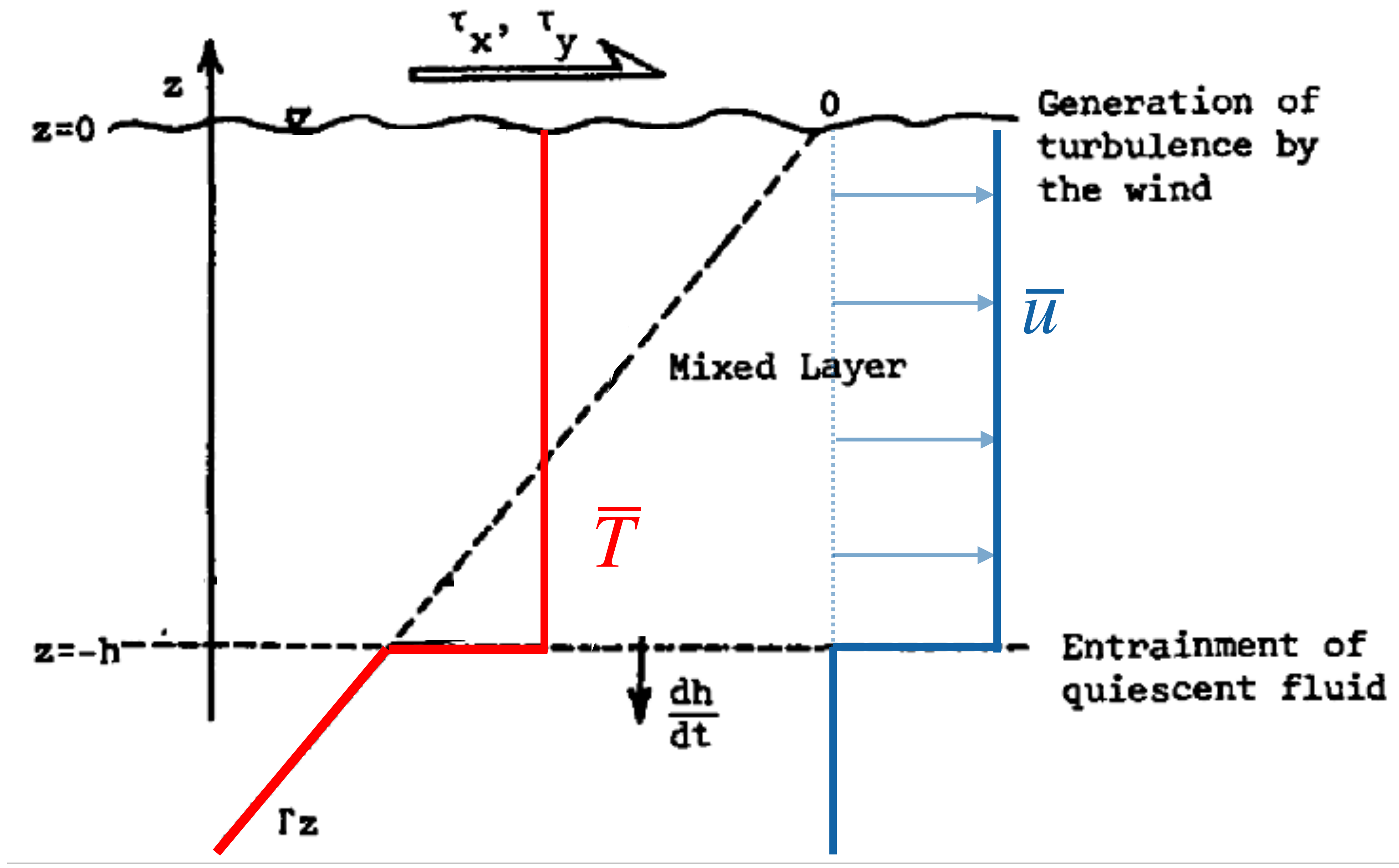
Olivier Marchand

- **Heated floor [290-353] kW**
- **Inner cylinder (5m)**
- **Temperature probes**
  - 3 Vertical profilers
  - 2 Fixed probes (z =0; 12cm)
- **Vertical laser sheet (30x25)cm**
  - PIV Stereo
- **Horizontal laser sheet (3x4)m**
  - PIV (z = 10cm)
  - PIV in volume (multi- layer)
- **IR camera (3x4)m**





Longer time behavior of the ML: Slab model (Pollard et al, 1973)



- Uniform moving layer:

$$\frac{d\langle u\rangle}{dt} - f\langle v\rangle = u_*^2 \qquad \frac{d\langle v\rangle}{dt} + f\langle u\rangle = 0.$$

$$\longrightarrow \quad \langle u\rangle = \frac{u_*^2}{f}\sin(ft) \qquad \langle v\rangle = -\frac{u_*^2}{f}(1-\cos(ft))$$

$$\longrightarrow \quad E_{slab} = \left[\langle u\rangle^2 + \langle v\rangle^2\right]/2 \;=\; \frac{u_*^4}{f^2}(1-\cos ft)$$

$$\frac{dE_{slab}}{dt} = u_*^2\ \langle u\rangle$$

- Potential energy: (from geometrical considerations)

$$E_{pot} = -\int_{-H}^{0} bz dz = -\int_{-H}^{-h} N_0^2 z^2 dz + N_0^2\frac{h}{2}\int_{-h}^{0} z dz = N_0^2\left[\frac{h^3 - H^3}{3} - \frac{h^3}{4}\right]$$

- Kinetic energy: (Since $E_{kin} = E_{slab}/h$)

$$\frac{dE_{kin}}{dt} = \frac{dE_{slab}}{h\,dt} - \frac{dh}{h^2\,dt}E_{slab}$$

- Equation for energy:

$$\frac{dE_{kin}}{dt} + \frac{dE_{pot}}{dt} = u_*^2\ \langle u\rangle/h \quad\longrightarrow\quad -E_{slab}\frac{dh}{h^2 dt} + \frac{dE_{pot}}{dt} = 0$$

$$\left[-\frac{u_*^4}{h^2 f^2}(1-\cos ft) + N_0^2\frac{h^2}{4}\right]\frac{dh}{dt} = 0 \quad\longrightarrow\quad \boxed{\frac{dh}{dt} = 0 \quad \text{or} \quad h^4 = \frac{4u_*^4}{f^2 N_0^2}(1-\cos ft)}$$

Longer time behavior of the ML: Extended Slab model

- Slab energy: $$\frac{dE_{slab}}{dt} = u_*^2 \ \langle u \rangle$$
- Potential energy: $$E_{pot} = \ N_0^2 [\frac{h^3 - H^3}{3} - \frac{h^3}{4}]$$
- Kinetic energy: $$E_{kin} = E_{slab}/h + \boxed{u_*^2 \tilde{E}_{(h)}}$$

$\tilde{E}_{(h)}$ expresses the kinetic energy associated with the deviation of the velocity from the uniform slab velocity.

$$\tilde{E} = \frac{1}{2u_*^2} \int (\mathbf{u} - \frac{\langle \mathbf{u} \rangle}{h})^2 dz = \frac{1}{u_*^2} \left[E_{kin} - \frac{u_*^4}{f^2 h}(1 - cos(ft) \right]$$

- Exact equation for energy:

$$\left[\underbrace{-\frac{u_*^2}{h^2 f^2}(1 - \cos(ft)) +}_{\text{SLAB}} \underbrace{\frac{N_0^2}{u_*^2}\frac{h^2}{4}}_{\text{POTENTIAL}} \right] \frac{dh}{dt} = \boxed{\overbrace{\underbrace{\left(u_{(z=0,t)} - \frac{\langle u \rangle}{h} \right)}_{\text{EXTRA PROD}} - \underbrace{\frac{\mathcal{E}}{u_*^2}}_{\text{DISSIPATION}} + \underbrace{\frac{d}{dt}\tilde{E}}_{\text{DEVIATION}} + \underbrace{\frac{d}{dt}\tilde{E}_{turb}}_{\text{IMBALANCE}}}^{=\mathcal{R}\ >0}}.$$

- Residual terms: $$\mathscr{R} \simeq m_p \frac{u_*^4}{hf}$$

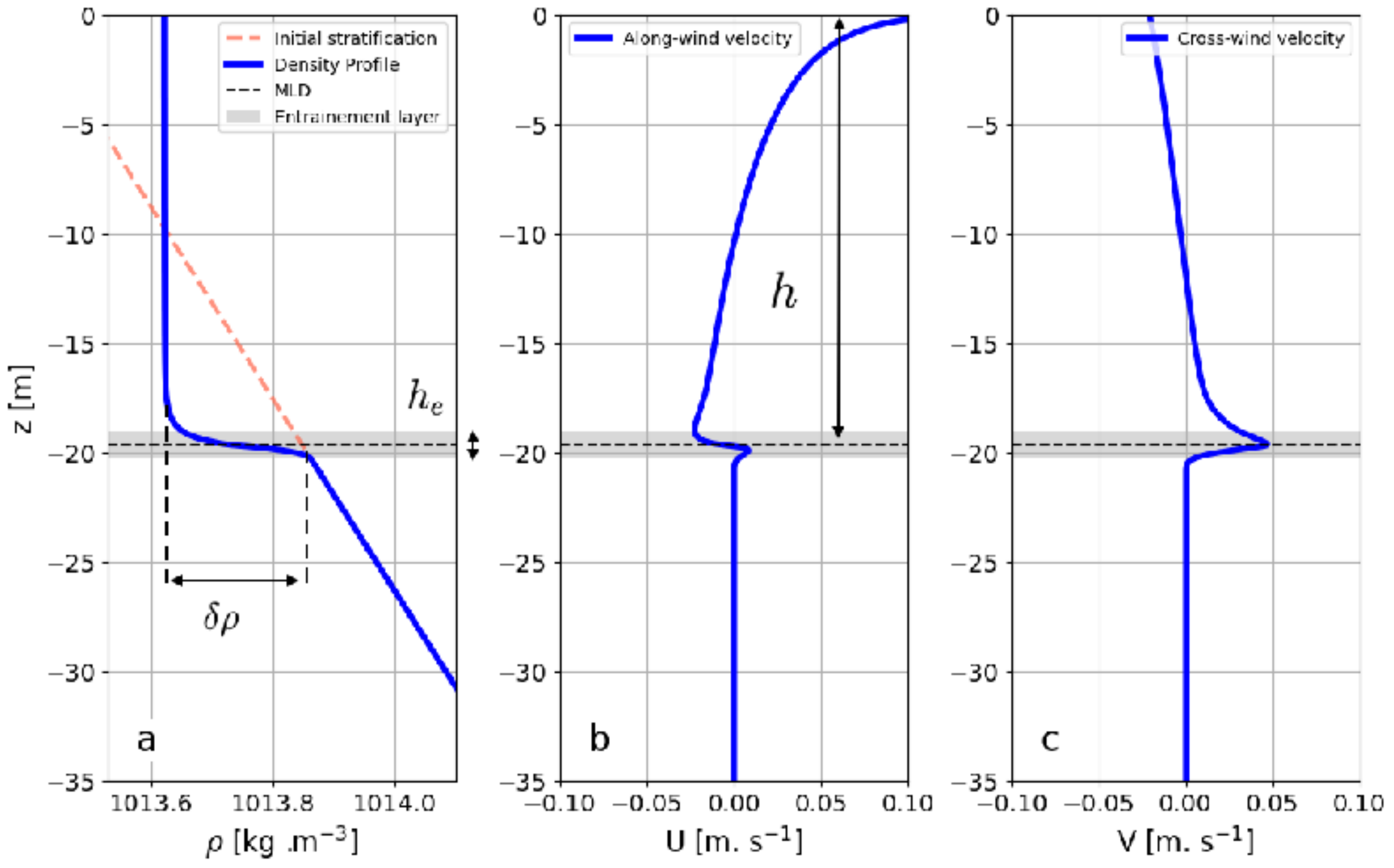
$\mathscr{R}$ is a fraction m_p of the turbulent kinetic energy production in the entrainment layer

$$\left[-\frac{u_*^4}{f^2 h^2}(1 - \cos(ft)) + N_0^2 \frac{h^2}{4} \right] \frac{dh}{dt} = m_p \frac{u_*^4}{hf}$$

Consistant with scaling from LES

$$\boxed{h^3 \frac{dh}{dt} = 4 m_p \frac{u_*^4}{N_0^2 f}}$$

Longer time behavior of the ML: Entrainment layer model



$$\left[-\frac{u_*^4}{f^2h^2}(1-\cos(ft)) + N_0^2\frac{h^2}{4}\right]\frac{dh}{dt} = m_p\frac{u_*^4}{hf}$$

$$\boxed{h^3\frac{dh}{dt} = 4m_p\frac{u_*^4}{N_0^2 f}}$$

- Considering a fully mixed layer and an entrainment layer h_e

  Mixing occurs only in the entrainement layer $\longrightarrow$ Marginal stability

Buoyancy drop	Velocity drop (Averaged over T_f)
$\partial b/\partial z \simeq N_0^2 h/(2h_e)$	$(\partial u/\partial z)^2 \simeq 2u_*^4/(f^2h_e^2)$

- Relation Between active/entrainment layer thickness:

$$\frac{h_e}{h} = 4Ri_{st}\frac{u_*^4}{h^4N_0^2f^2}$$

- Local Buoyancy: $N_e^2 = N_0^2 h/(2h_e)$

- Turbulent Viscosity: (From dimensional argument)

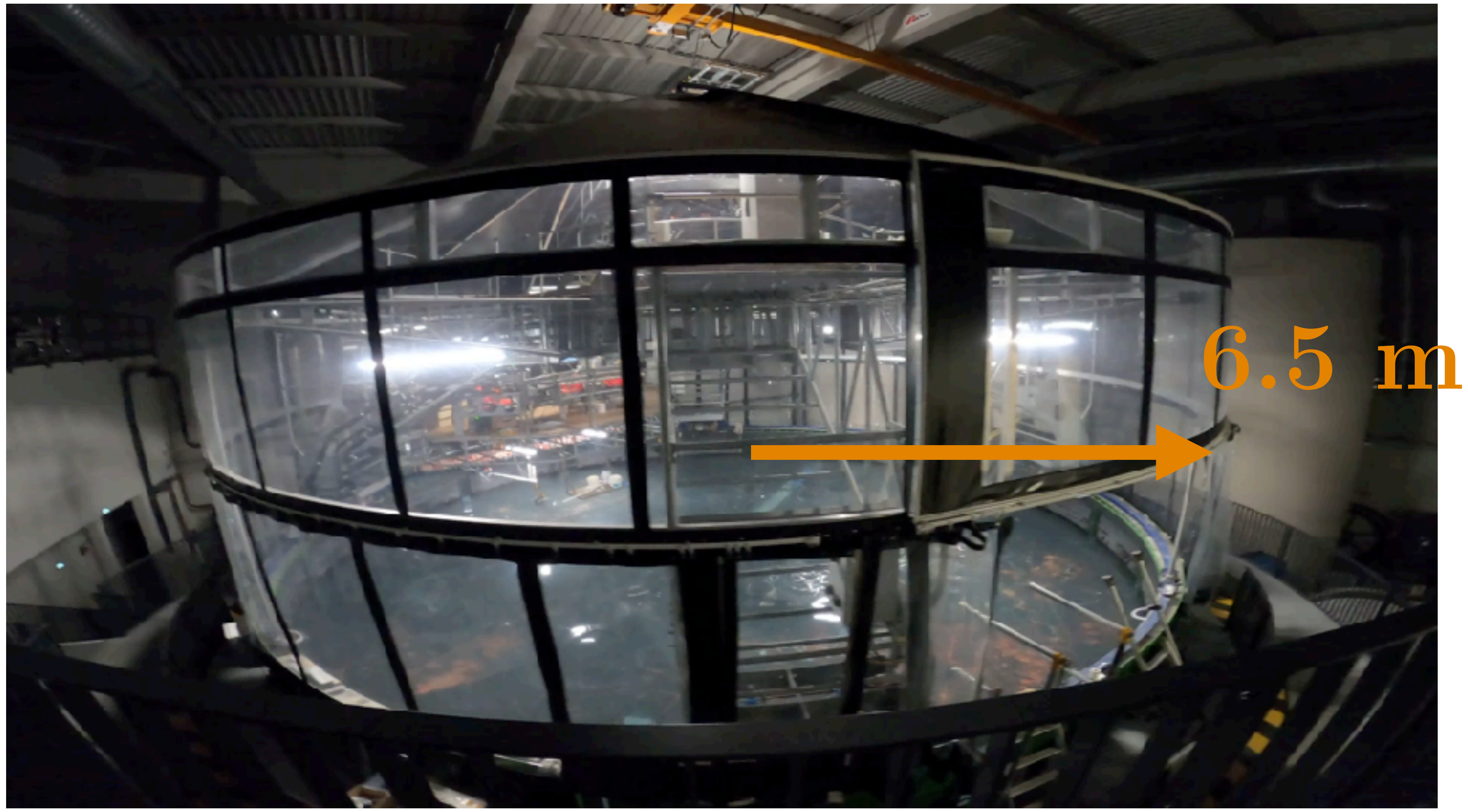
$$\nu_t \simeq u_*^2/N_e \simeq 2\sqrt{2}Ri_{st}^{1/2}\frac{u_*^4}{h^2N_0^2f}$$

- TKE Production: (integrated over the layer) $< P >_e \simeq \nu_t u^2/h_e$

$$u^2 \simeq 2u_*^4/(f^2h^2)$$

- Mixing efficiency $dE_p/dt \propto \frac{u_*^4}{hf}m_p < P >_e$

Build upon Kato-Phillips experiments



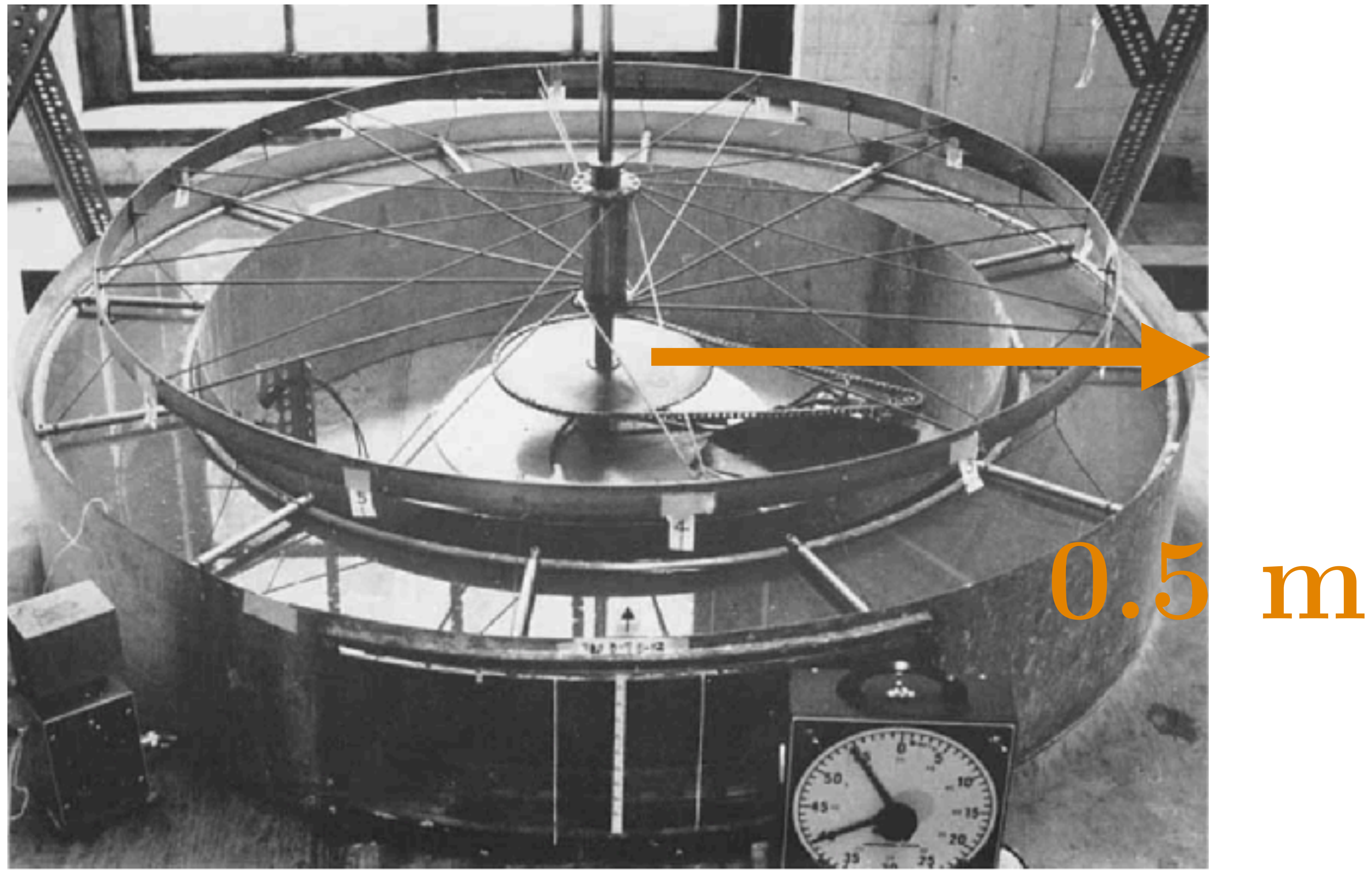


FIGURE 1. The experimental apparatus.

- Fine control over the rotation
- Larger scale (More turbulent / smaller curvature)

Kato - Philips 1969:

- Impulsive rotation of the upper screen
- Stratified in salt

Seminal Experiments

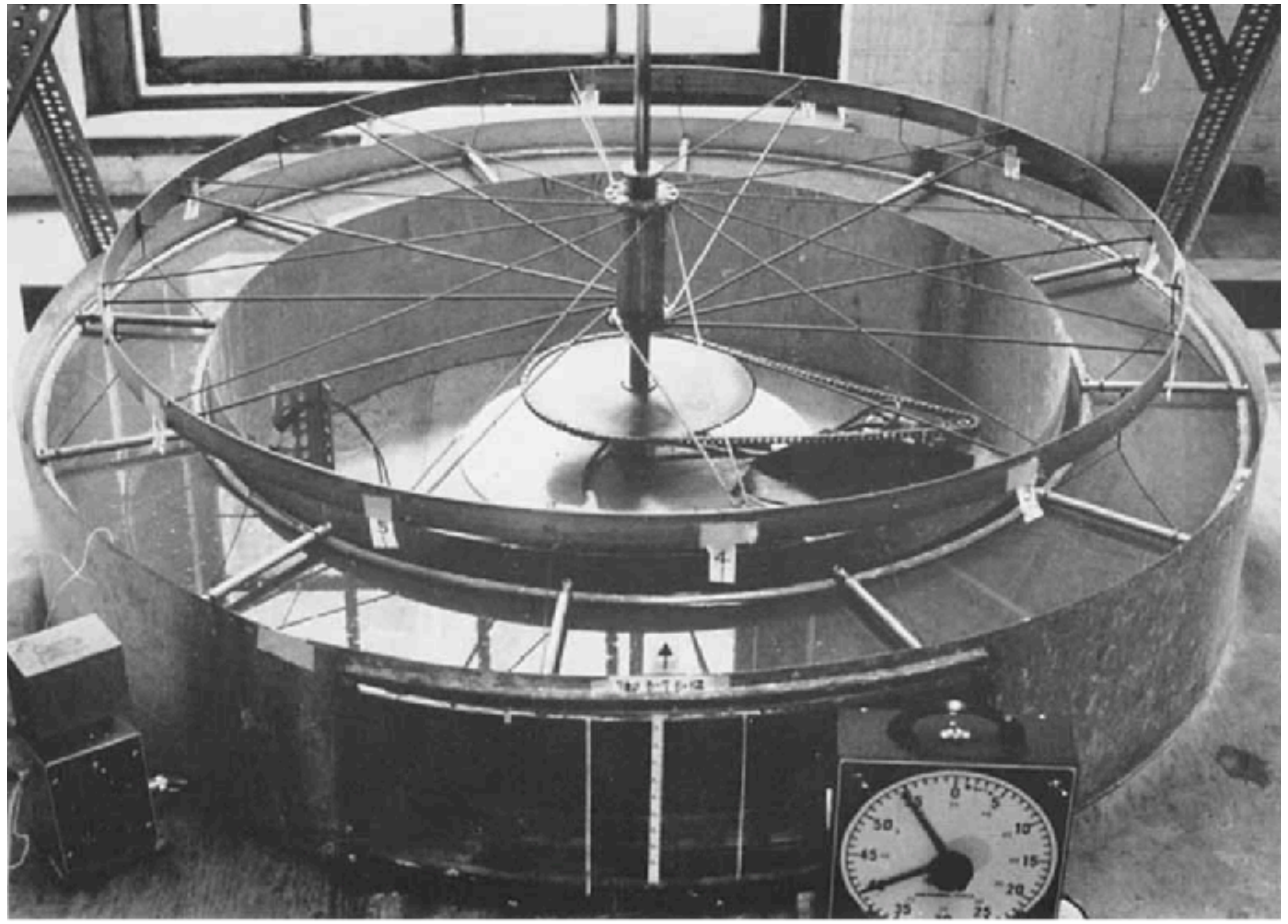


FIGURE 1. The experimental apparatus.

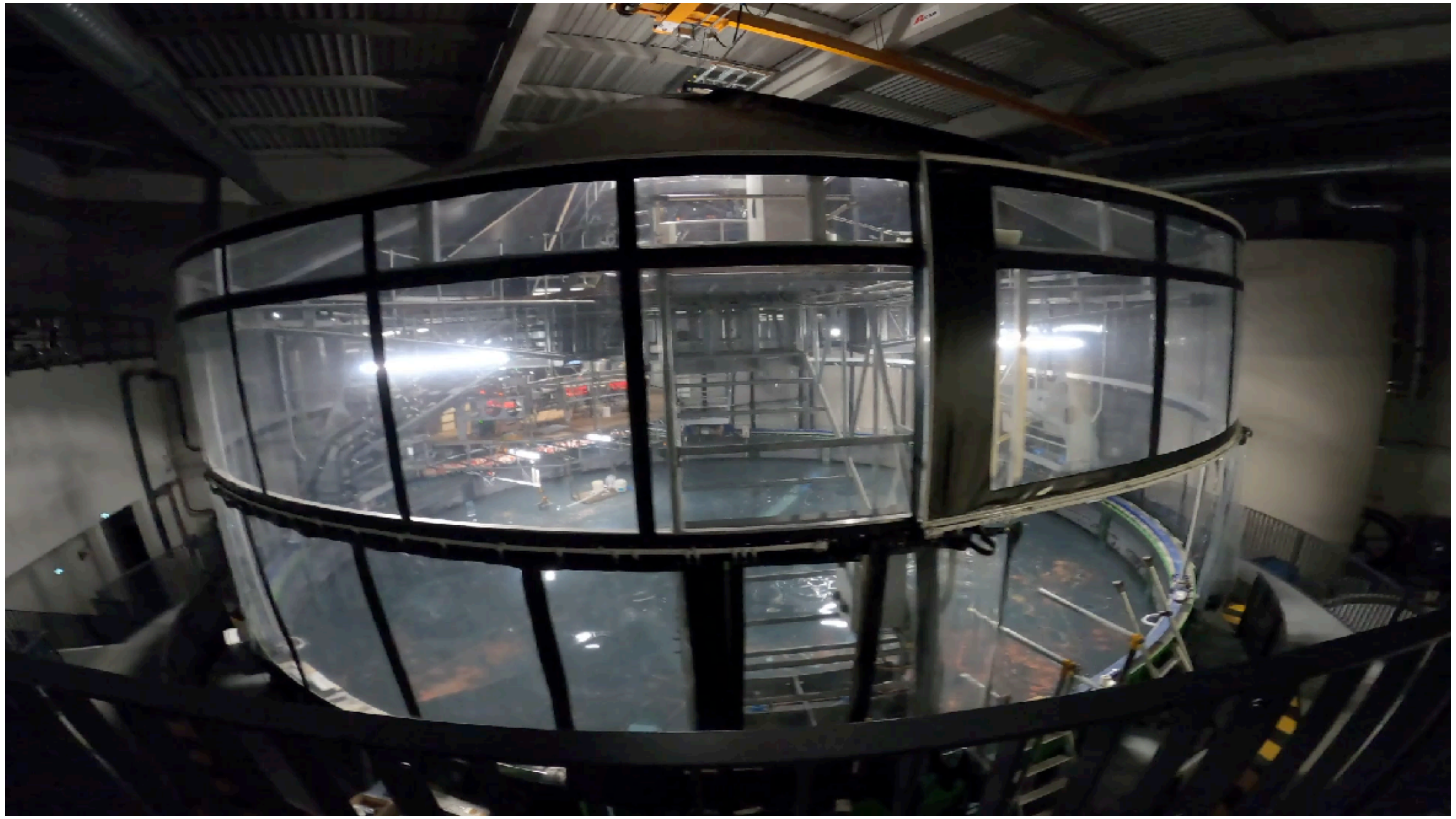
Kato - Philips 1969

- Mixed layer deepening rate : $h \sim t^{1/3}$
- Entrainment law : $E(Ri) = \dfrac{dh/dt}{u_*} = 2.5\,Ri^{-1}$

- They did not considered rotational effects
- The torque applied was tuned « by hand »
- No direct measurement of the density
- No direct measurement of the velocity

Kato, H., Phillips, O.M., 1969. On the penetration of a turbulent layer into stratified fluid. Journal of Fluid Mechanics 37, 643–655

Coriolis Platform

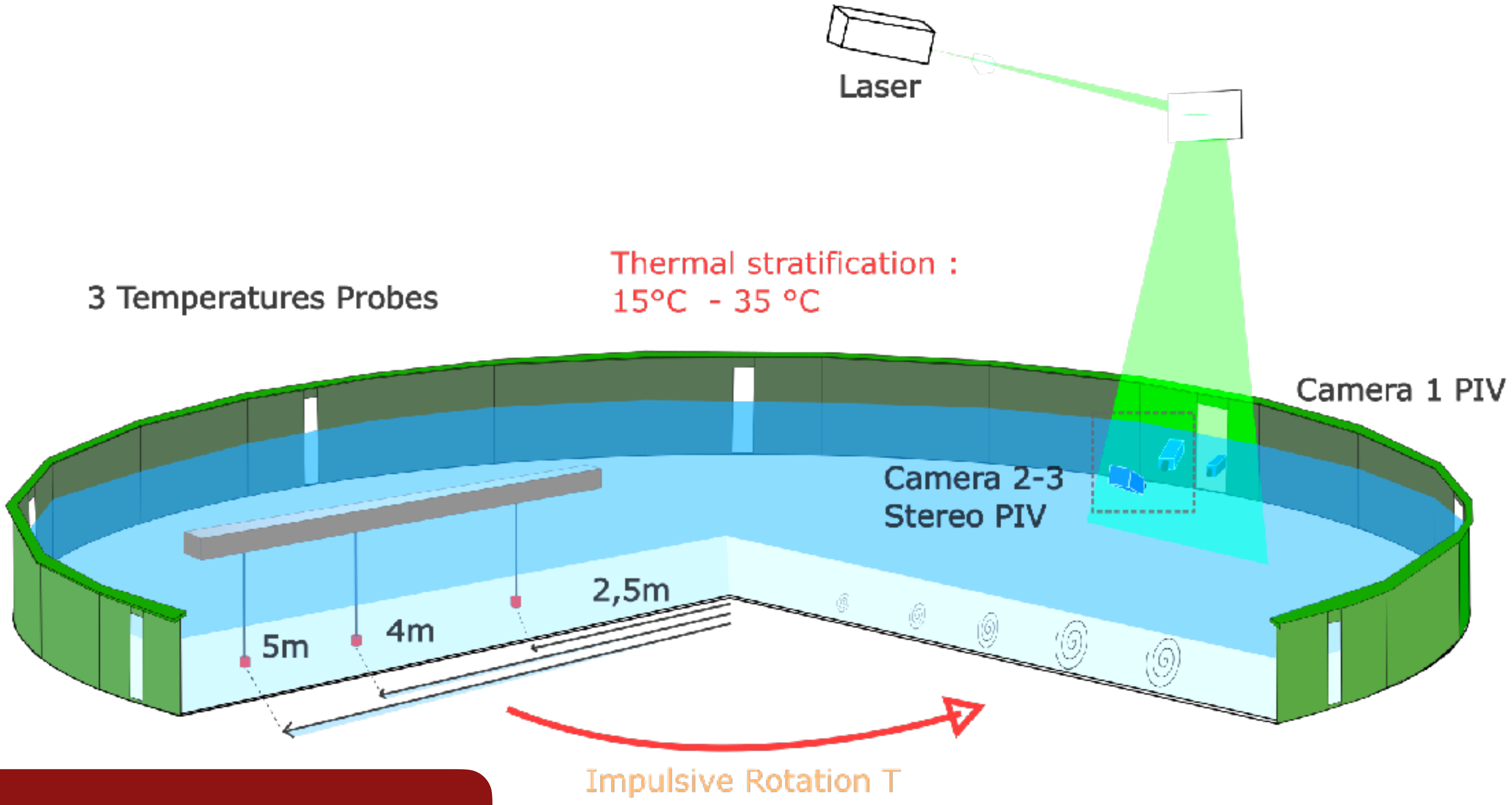


- **Diameter:** $13\ m$
- **Weight :** 350 Tones at full load
- **Maximum Speed:** $6\ rpm$
- **Max water height:** $1\ m$
- **Volume:** $132\ m^3$

- **Rossby Number** $Ro = \dfrac{U}{fL}$
- **Froude Number :** $Fr = \dfrac{U}{NL}$
- **Reynold Number:** $Re = \dfrac{UL}{\nu}$

Forced Convection Experiment: Apparatus

- **Acceleration of rotation (Spin-Up)**
- **Temperature stratification**
- **Temperature probes**
  - 3 Vertical profilers
- **Vertical laser sheet (30x25)cm**
  - PIV Stereo (2D - 3 components)



Control parameters

Friction : u_* **Rotation :** f **Stratification** $N^2 \equiv (\Delta T)$